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Research Article

APPLICATION OF A LOGISTIC FUNCTION TO DESCRIBE THE GROWTH OF FODDER GALEGA

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ABSTRACT

It was attempted in the paper to describe the growth of oriental goat's rue (*Galega orientalis* Lam.) growth by means of a logistic function. The studies demonstrated that the function describes empirical data well as the coefficients of determination obtained ranged between 97 to 98%, depending on the growing season. The estimated function parameters indicate that both the plant height and growth rate were conditioned by thermal conditions and moisture during the growing season. Plants had the longest stems in the year 2010 (127.9 cm), and the shortest in 2008 (105.9 cm). The maximum growth rate ranged from 2.12 to 3.97 cm per day in, respectively, 2009 and 2008. Also the inflection point, the point at which plants grew the fastest, depended on the conditions of the growing season.

Keywords: oriental goat's rue, logistic function, growth equation, growth rate equation, growth rate.

INTRODUCTION

Oriental goat's rue (*Galega orientalis* Lam.) is a plant of the family *Fabaceae* Lindl., subfamily *Fabaoideae,* tribe *Galegae* and subtribe *Galeginae*. It is only recently that scientists worldwide [Šlepetys, 2003; Baležentienė, 2009, 2011; Egamberdieva et al., 2010; Mikkonen et al., 2011] and in Poland [Deska, 2009; Deska et al., 2012, Kozłowski and Zielewicz, 2013] have started to be interested in the plant. Indigenous to the northern Caucasus, the plant can be found in the wild in forested regions of Georgia, southern Armenia, south-western Azerbaijan and sometimes in the Crimea. Due to the fact that the plant's green parts are rich in protein (the content ranges between 16–27%), oriental goat's rue is commonly grown in some EU Baltic countries (Estonia, Finland, Latvia) and in Russia. In Poland the species is relatively unpopular, just like in France, Canada, Croatia, Belarus and Serbia where the first attempts are being made to introduce the plant on a wider scale. Studies on the biology of growth and morphological development of the oriental goat's rue, conducted by Raig [2001], Ignaczak [2010] have demonstrated that the development of the plant is a complex process. During the growing season, leaf rosettes are formed on root necks in early spring. When the minimum temperature exceeds 5^{o}C, plants start to produce aerial stems from root necks that have overwintered, and one-year-old underground roots. During blossoming and pod maturation, cluster tips wither and shrink if rainfall is insufficient. Under the climatic conditions of Poland, pods in main clusters mature in late July. However, if late July is rainy, oriental goat's rue's plants develop secondary and tertiary stems which produce flowers [Deska and Wyrzykowska, 2005; Deska, 2009; Deska et al., 2012].

Similarly to any other plant, the development of oriental goat's rue can be described mathematically by means of indices and functions. The functions that describe the relationship: age-time, are called growth curves and make it possible to include many measurements into one equation based on several parameters which can be interpreted from a biological standpoint. They are sigmoid curves consisting of three stages: "exponental" characterized by slow but constantly increasing growth, "linear" when the growth is the fastest till maximum is reached, and the stage of "growing old" when the growth slows [Gregorczyk, 1991; Wesołowska-Janczarek, 1993; Karadavut et al., 2008]. Analysis of plant growth frequently employs the logistic function – a function which is continuous and differentiable at each point in its domain, with an inflection point with respect to which the graph is symmetrical [Tsoularis, 2001; Tsoularis and Wallace, 2002].

The mathematical description of a completed process may contribute to enhanced knowledge of the biology of plant development, indicate moments of optimum agrotechnological actions, help to forecast yields as well as indicate morphological and developmental differences of individuals and the whole population.

The objective of the work was to examine the growth of oriental goat's rue *Galega orientalis* Lam.) during a 4-year period by means of a logistic function.

MATERIALS AND METHODS

Analysis of data was based on measurements of stem lengths of oriental goat's rue (*Galega orientalis* Lam.) cv. Gale which were taken during a four-year period as part of a long-term experiment set up in 2007 at the University of Natural Sciences and Humanities experimental site (52º10'N, 22º17'E Poland).

From 2007 to 2010, 70 plants were randomly selected every seven days from the start of growth to the stage of full maturity of pods. The beginning of growth was assumed to be the moment of forming leaf buds on root necks of the plants which overwintered. Thermal conditions and rainfall during the growing seasons are presented in Figures 1 and 2.

The temperature varied in all the study years. Relatively cool spring of 2007 saw a sudden increase in temperature in late April, and high temperatures at the end of the growing season. In the years 2008 and 2010 the beginning of the growing season was characterized by quite low tem-

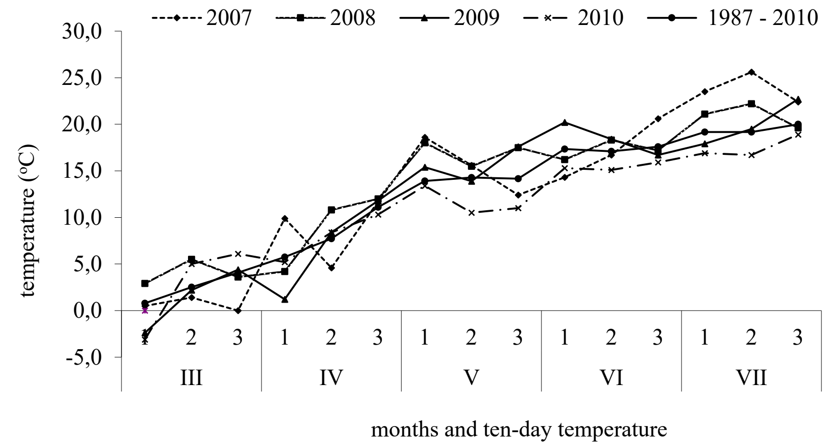

Figure 1. Ten-day and monthly temperatures (ºC) in the study years

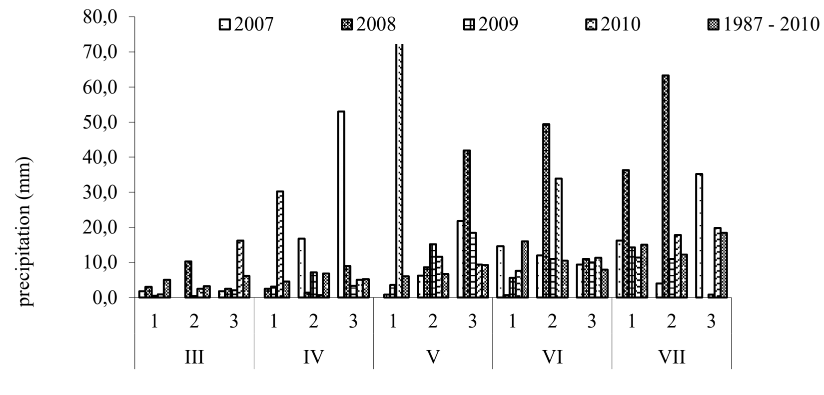

Figure 2. Ten day and monthly precipitation (mm) in the study years

peratures, around 5°C, but later they increased steadily. The temperature in the year 2009 was the lowest in the spring and was still around 1°C as late as in early April. Then a sudden increase in temperature was observed.

The worst hydrological conditions were recorded in 2007 and 2009 when the precipitation did not exceed 200 cm during the growing season. Moreover, it was unevenly distributed over time. With this respect, the best year was 2010 when the high rainfall was also evenly distributed and, as a result, met the needs of plants.

The following function was used to describe the growth of oriental goat's rue [Richards, 1969]:

$$y(t) = \frac{a}{1 + b \cdot e^{-k \cdot t}}$$

where: $y(t)$ – denotes plant's height on day t,
a – the value of asymptote,
b – an integration constant and
k – the index of growth rate.

The terms a, k and b are somehow interdependable. If k values are high, a values are low and describe small plants. By contrast, low k values generate high values, which is typical of high plants. Values of the parameter b depend on initial values and, in general, higher plants have got lower b values.

Equations of functions of average growth were estimated for each year separately. The criterion of goodness of fit was the determination coefficient R^2, which is frequently used for this purpose, calculated following the formula [Seber and Wild, 1989; Ryan, 1997]:

$$R^2 = 1 - \frac{SSE}{SST}$$

in which:

$$SSE = \sum_{i=1}^{n} (y_i - f_i)^2, \quad SST = \sum_{i=1}^{n} (y_i - \bar{y})^2,$$

where: y_i – experimental data;
f_i – values of the function fitted corresponding to the values y_i,
$\bar{y}$ – arithmetical mean of the experimental data.

Values of the coefficient are close to 1 when the fit is good, and they are close to 0 when the fit is poor. If a function examined completely disagree with the experimental data, the coefficient may be negative, which indicates that the function should be discarded because it is not the appropriate model to describe the process [Bochniak and Wesołowska-Janczarek, 2006].

Moreover, the equation of growth rate was calculated in addition to the theoretical initial value of growth and inflection points, according to the formulae [Gregorczyk, 1991]:

- equation of growth rate:

$$\frac{dy}{dt} = \frac{k}{a} \cdot y(t) \cdot (a - y(t))$$

- theoretical initial value of growth:

$$y_0 = \frac{a}{1 + b}$$

- coordinates of the inflection point of the function:

$$t_i = \frac{\ln b}{k}, \quad y(t_i) = 0{,}5 \cdot a$$

In order to compare the growth patterns in individual years, the function was linearized following the formula: lnb – k×t. Next, the hypothesis was checked whether or not the slopes of strait lines are equal so as to find out if the course of the straight lines examined is the same. The hypothesis $H_0 = k_1 = k_2 = k_3 = k_4$ was checked using Tukey's test following Elandt [1964].

All the calculations were performed using Statistica 9.0 and the Excel spreadsheet.

RESULTS

Figure 1 shows the growth of goat's rue in the study years. It demonstrates that the growth was the best in 2010 when precipitation distribution coincided with the water needs of plants. Also function parameters, describing growth equations and growth rate equations, confirm this finding (Table 1, 2 and Figure 3). In 2010 the plants had the highest theoretical asymptotic value (parameter a) and low values of coefficients k and b.

In 2010 oriental goat's rue plants as early as after 42 days of growth reached the inflection points at the stem length of 64 cm and the maximum growth rate of 3.45 cm·day^{-1}. Similar growth was observed in the year 2008 when stems had the lowest theoretical initial value (0.131 cm) and asymptotic maximum length (105.95 cm). However, conditions during the growing season (rainfall met the water needs of plants) resulted in the inflection point which was by three days longer compared with 2010, the growth rate being the highest as it was almost 4 cm per day.

Table 1. Maximum empirical value reached by plants and logistic function parameters which approximate the elongation of goat's rue plants in the study years

Year	y_{max} (cm)	y_o (cm)	Function parameters			t_i (day)	$y(t_i)$ (cm)	$\frac{dy}{dt}$ (cm·day^{-1})
			a	b	k			
2007	108.0	0.268	107.7	2.95×10^{2}	1.11×10^{-1}	51.7	53.8	2.96
2008	108.1	0.131	105.9	8.06×10^{2}	1.50×10^{-1}	44.6	52.9	3.97
2009	104.9	3.04	106.0	5.09×10^{1}	0.80×10^{-1}	49.1	53.0	2.12
2010	129.1	1.38	128.0	9.19×10^{1}	1.11×10^{-1}	41.9	64.0	3.45

y_{max} – empirical maximum growth, y_o – theoretical initial growth, t_i, $y(t_i)$ – coordinates of inflection point, $\frac{dy}{dt}$ – maximum growth rate

Table 2. Goat's rue growth equations and growth rate equations in the study years

Years	Growth equation	R^2	Growth rate equation
2007	$y = 107.7\times [1+2.95\times10^{2}\times\exp(-1.11\times10^{-1}t)]^{-1}$	0.998	$dy/dt = 10.3\times10^{-4}\times y(t)\times[107.7-y(t)]$
2008	$y = 105.9\times [1+8.06\times10^{2}\times\exp(-1.50\times10^{-1}t)]^{-1}$	0.997	$dy/dt = 14.2\times10^{-4}\times y(t)\times[105.9-y(t)]$
2009	$y = 106.0\times [1+5.09\times10^{1}\times\exp(-0.80\times10^{-1}t)]^{-1}$	0.998	$dy/dt = 7.8\times10^{-4}y(t)\times[106.0-y(t)]$
2010	$y = 127.9\times [1+9.19\times10^{1}\times\exp(-1.11\times10^{-1}t)]^{-1}$	0.998	$dy/dt = 8.7^{-4}\times y(t)\times[127.9-y(t)]$

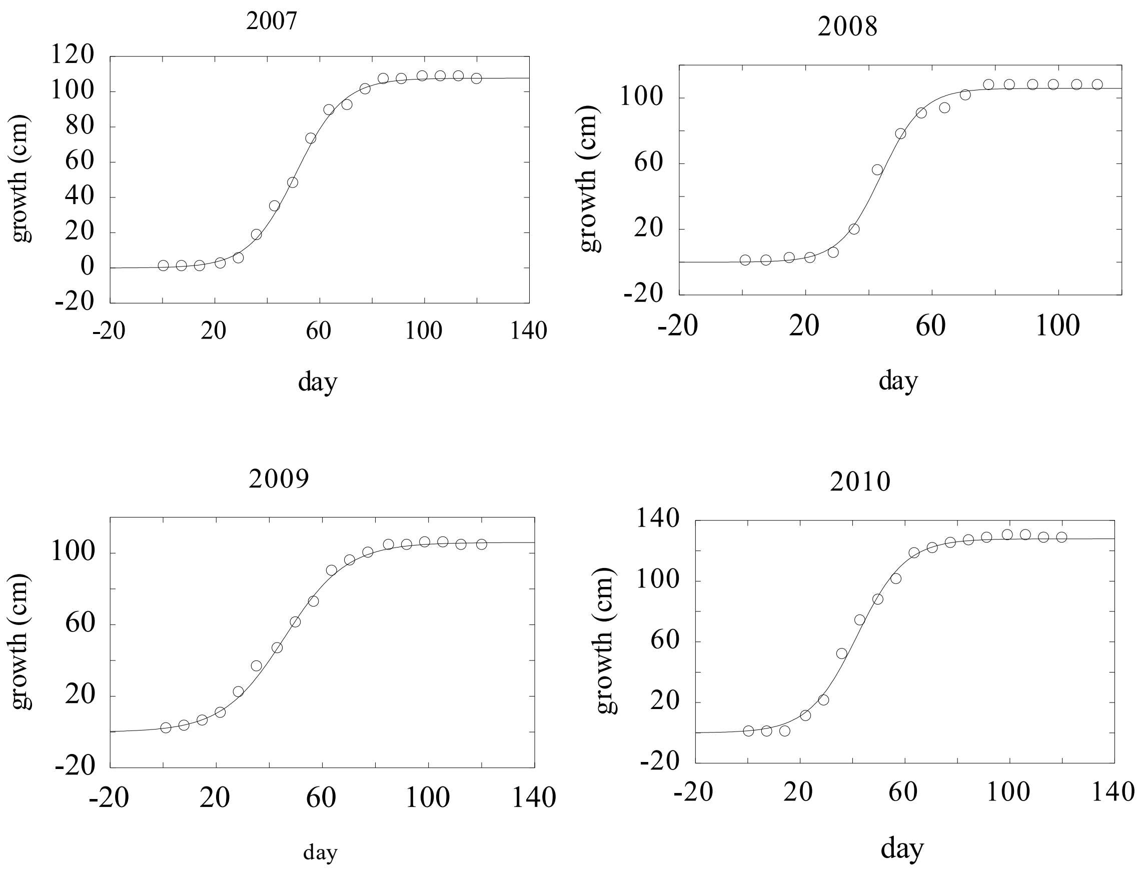

Figure 3. Fit of estimated functions to the empirical data in individual study years

The year 2009 was the least beneficial for goat's rue plants as it was characterised by unfavourable thermal and water conditions (the precipitation sum did not exceed 200 mm). Low temperature for a long time at the start of growth did not stimulate the growth of goat's rue. Over the four-year period plants had the lowest empirical length of stems – 104.86, the lowest growth index k and the lowest theoretical maximal growth rate (2.12 cm in the 79th day of growth and at the stem length of 53 cm) (Table 1, 2). The plants of goat's rue reached the inflection point as late as on the 52th day of growth and height of 54 cm. It was associated with maximum growth rate of almost 3 cm per day (Table 1, 2, Figure 4).

After linearization different lines were obtained, which was statistically confirmed. The value of F (9.40) was higher than the critical value F (3.13) so the hypothesis assuming equal regression linear coefficients (k) was refuted and the alternative hypothesis was accepted, which meant that the lines were not parallel (Figure 5).

The function applied is a good representation of the process of rue's plant growth, as indicated by high determination coefficients as well as figures demonstrating how well the estimated curves fit the empirical data (Table 2, Figure 6).

DISCUSSION

Plant growth is a complicated process which can be described by suitable mathematical functions. So far, many mathematical models were developed which reflect the growth process of plants and their individual organs. Curry [1971] as well as Curry and Chen [1971] suggested a mathematical growth model of maize roots, stem and leaves. Baker et al. [1972] and Pyda [1977] developed the growth model of, respectively, cotton and rape plants. As science progresses, the already existing models are being replaced by new ones which produce more and more reliable values (better reflect real values) [France, 1984]. Growth processes may be described, similarly to this and other works Seidler and Gregorczyk [1986], Mustears [1989], Villegas et al. [2001], Damgaard et al. [2002], by the logistic function. Also Karadavut et al. [2008] used the logistic function to develop the model of wheat growth. As the logistic function fits empirical data well in this work, it may indicate that the function can be used to describe the growth and development of goat's rue. Also Damgaard and Weiner [2008] and Karadavut [2009] reported a good fit of logistic curve to empirical data. An appropriately

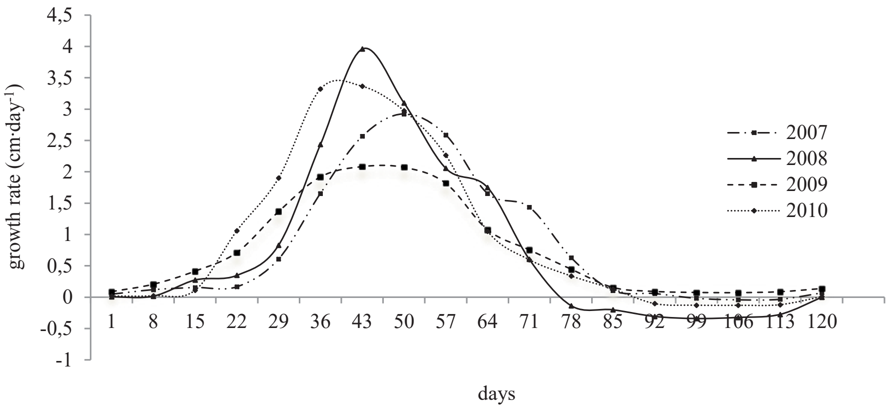

Figure 4. Theoretical equations of growth rate in the study years

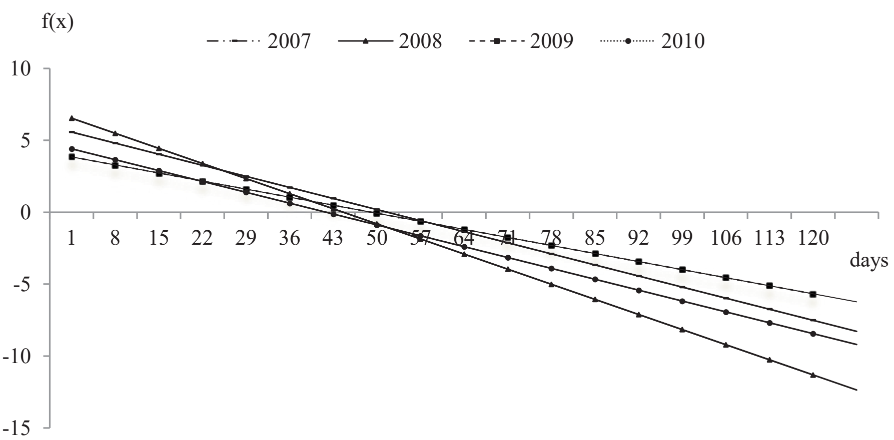

Figure 5. Plant growth equations in the study years (following linearization)

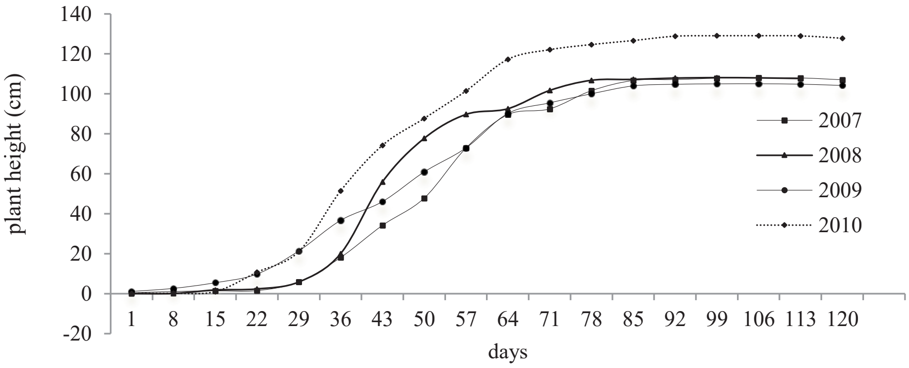

Figure 6. Empirical plant growth in individual study years

adjusted model (mathematical function) makes it easy to compare experimental data and sometimes enables forecasting plant growth.

Similarly to Gregorczyk [1991], there was observed varied dynamics of plant elongation in individual years, which was indirectly supported by the statistical comparison of lines of the estimated functions. This indicates that weather conditions had a marked impact on the growth of goat's rue which requires quite a lot of water due to abundant canopy, similar findings reported by Patterson [1993]. The maximum theoretical growth rates calculated in this work ranged between 2.12 and 3.97 cm per days in 2009 and 2008, respectively. Also inflection points were different in individual years. The longest inflection point was observed in 2009 and the shortest in 2010.

CONCLUSIONS

1. The logistic function enables a good approximation of theoretical data to empirical data, which is supported by high values of determination coefficients describing how well the growth equations determined fit the experimental data.
2. Weather conditions in the individual study years differently influenced the growth and development of goat's rue, as indicated by growth function parameters, growth rate equations and linearized growth equations.
3. Asymptotic growth values ranged from 105.9 to 127.9 cm, depending on the growing season, and, in general, they were lower than the maximum empirical values.
4. In individual years goat's rue plants differed as to the maximum theoretical rate of growth and the date at which the inflection point was observed. The extreme values were, respectively 1.85 cm per day and 10 days. In 2010, the inflection point was observed on the 42th day, and in 2007 on the 52th day of growth. Due to the conditions of the year 2008 (rainfall which met plants' needs), the theoretical growth rate of goat's rue was the greatest and reached 4 cm per day whereas in 2009 the maximum growth per day was slightly above 2 cm.

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