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Research Article

COMPARISON OF DETEMINISTIC INTERPOLATION METHODS FOR THE ESTIMATION OF GROUNDWATER LEVEL

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ABSTRACT
This paper compares two spatial interpolation techniques – Radial Basis Functions (RBF) and Inverse Distance Weighting (IDW) – with the goal of determining which method creates the best representation of reality for measured groundwater levels in catchment area. The study used the results of research and field observations from the year 2011, in Sosnowica (West Polesie). The data set consists of groundwater levels measured at 15 points in three series of tests. Surface generation was obtained for each method. The water prediction maps showed spatial variation in the groundwater level in the study area and they are quite different. RBF method resulted in a smoother map. The analysis of the methods of interpolation of analyzed data with the help of cross validation statistics and plots showed that Radial Basis Functions creates better representation of reality for measured groundwater levels.

Keywords: Radial Basis Functions, Inverse Distance Weighting, groundwater levels, prediction maps.

INTRODUCTION

Interpolation is a method or mathematical function that estimates the values at locations where no measured values are available. Spatial interpolation assumes the attribute data are continuous over space. This allows the estimation of the attribute at any location within the data boundary. Another assumption is that the attribute is spatially dependent, indicating the closer values are more likely to be similar than the values farther apart. The goal of spatial interpolations is to create a surface that is intended to best represent empirical reality thus the method selected must be assessed for accuracy [1].

There is no single preferred method for data interpolation. Aspects of the algorithm selection criteria need to be based on the actual data, the level of accuracy required, and the time and computer resources available. Selecting an appropriate spatial interpolation method is fundamental to surface analysis since different methods of interpolation can result in different surfaces and ultimately different results.

This paper compares two spatial interpolation techniques – Radial Basis Functions (RBF) and Inverse Distance Weighting (IDW) – with the goal of determining which method creates the best representation of reality for measured groundwater levels.

MATERIALS AND METHODS

The drainage area of the ditch K-2 discharging surface waters to the Peony river was selected to study the variability of water. It is located in the Sosnowica village in West Polesie [6]. Ditch drainage area is 0.46 km^2 and is 86% used as a once-semi-natural grasslands, the remaining 14% are birch and pine woodlands. The basin are 75% moorshed and moist habitats characterized by high levels of ground water and the position of small variations in retention. The catchment area of the trench has a very small decrease of 1.1‰

and includes a flat bottom valley. In 2011, at 60 days in the vegetation grassland (30.03–31.10) at 15 points was measured depth of the groundwater table. The measurements were carried out in the middle distances drainage ditches in piezometric wells [10]. In that paper the data set consists of groundwater levels measured at 15 points in three series of tests: spring, summer and autumn.

On the run in the 2006–2009 the research shows that the depth of the water table depends on the size of evaporation and modified the effects of the hydrographic network. We recorded the smallest water depths position at the beginning of the growing season, while the largest depth of the water table was recorded in the height of summer. The most stable water levels occurred in the autumn [3].

There are two main grouping of interpolation techniques: deterministic and geostatistical. Deterministic interpolation techniques create surfaces from measured points, based on either the extend of similarity (Inverse Distance Weighted) or the degree of smoothing (Radial Basis Functions). IDW and RBF are exact interpolators, predict a value identical to the measured value at a sampled location [4].

Radial basis function methods are considered as exact interpolation techniques. The exact interpolators predict values identical with those measured at the same point and the generated surface requires passing through each measured points. The predicted values can vary above the maximum or below the minimum of the measured values [9]. There are five different basic functions: thin-plate spline, spline with tension, completely regularized spline, multiquadric function, and inverse multiquadric spline. Each function has a different shape and results in a different interpolation surface. While there are more entry points specified, the greater the influence of distant points and the smoother the surface [5]. The estimated values of the methods are based on a mathematical function that minimizes the total surface curvature, generating quite a smooth surface. The smoothness of the resulting surface is controlled by a smoothing parameter. Radial basis function are described in Bishop [2].

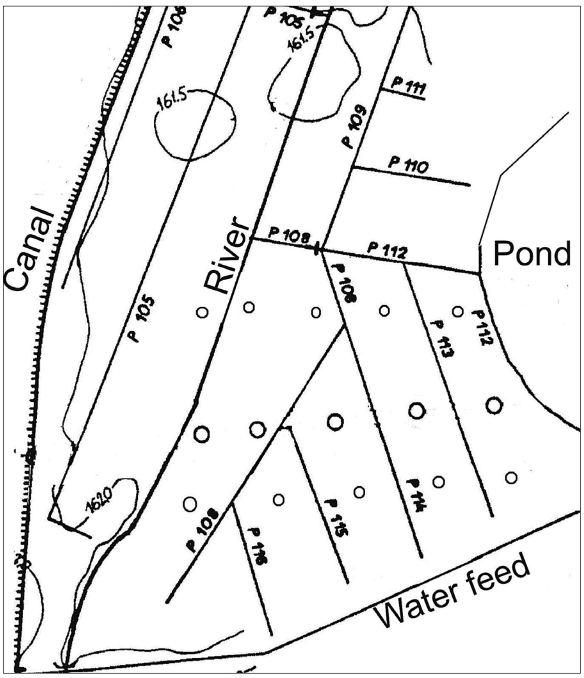

Figure 1. Location of measurement points groundwater level. o – points

Inverse Distance Weighting is based on the assumption that the nearby values contribute more to the interpolated values than distant observations. In other words, for this method the influence of a known data point is inversely related to the distance from the unknown location that is being estimated.

The general formula is:

$$\hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i)$$

where: $\hat{Z}(s_0)$ – predicted value for location s_0,
N – number of measured sample points surrounding the prediction location that will be used in the prediction
λ_i – weights assigned to each measured point
$Z(s_i)$ – observed value at location s_i.

The formula to determine the weights is the following:

$$\lambda_i = \frac{d_{i0}^{-p}}{\sum_{i=1}^{N} d_{i0}^{-p}}$$

where: p – arbitrary positive real number called the power parameter (typically $p = 2$),
d_{i0} – distance between the prediction location s_0 and each of the measured locations.
d_{i0} is given by:

$$d_{i0} = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}$$

where: (x_0, y_0) are the coordinates of the interpolation point s_0 and (x_i, y_i) are the coordinates of each dispersion point s_i.

The power parameter p influences the weighting of the measured location's value on the prediction of the location's value, that is, as the distance between the measured sample locations and the prediction location increase, the weight (or influence) that the measured point has on the prediction will decrease exponentially [4].

The adequacy of the fitted models was checked on the basis of validation tests. In this method, known as jackknifing procedure, interpolation is performed at all the data points, ignoring, in turn, each one of them one by one. The differences between estimated and observed values are summarized using cross-validation statistics [8].

For all points, cross-validation sequentially omits a point, predicts its value using the rest of the data, and then compares the measured and predicted values. The calculated statistics serve as diagnostics that indicate whether the model is reasonable for map production. In addition to visualizing the scatter of points around this 1:1 line (cross-validation scatter plot), a number of statistical measures can be used to assess the model's performance.

The differences between estimated and observed values are summarized using the cross-validation statistics: mean prediction error (ME), root-mean-square prediction error (RMSE). The summary statistics should meet the following criteria [7, 11]:

$$ME = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{z}(x_i) - z(x_i)\right) \cong 0;$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{z}(x_i) - z(x_i)\right)^2} \quad \text{minimum;}$$

where: $\hat{z}(x_i)$ – predicted value;
$z(x_i)$ – observed value;

A GIS software package ArcGIS 10 and ArcGIS Geostatistical Analyst extension were used for the interpolation methods in this study. The maps were produced with the ArcMap module of the ArcGIS.

RESULTS

Figure 2 and 3 shows the spatial distribution of groundwater level for analyzed three time series in the study area obtained by Radial Basis Functions (with the multiquadric function) and Inverse Distance Weighting (with power parameter $p = 2$). The prediction map provided by two interpolation methods (Figure 2 and Figure 3) are different. The comparison of IDW and RBF maps indicated that RBF method has resulted in a smoother map.

More quantitative comparison of these two techniques was obtained by comparing the cross-validation statistics (Table 1). The best model was selected based on two criteria: the mean prediction error (ME) nearest zero, the smallest root-mean-square prediction error (RMSE). IDW resulted in ME of -1.18 m to -2.18 m whereas RBF gave ME of -0.84 m to 0.46 m. Similarly, IDW gave RMSE of 11.90 m^2 to 17.44 m^2 and RBF 2.36 m^2 to 11.93 m^2. The ME values are closer to 0 and RMSE are smaller for RBF.

Table 1. Cross validation results for interpolation methods

Series	RBF		IDW	
	ME	RMSE	ME	RMSE
I	0.46	5.92	-2.18	16.37
II	-0.84	11.93	-2.21	17.44
III	-0.32	2.36	-1.18	11.90

The relationship between the interpolated values and the true observed data was also evaluated. Figure 3 presents the scatter plots of predicted versus measurement values obtained for used interpolation methods. It is expected that these should scatter around the 1:1 line. The cross-validation scatter plots provided by two interpolation methods (Figure 4) are different. The deviation from the 1:1 line are greater for the IDW method. It shows that within interpolation methods used, the RBF method is the one that best estimated the measurements results of the groundwater level.

CONCLUSIONS

In this study, two spatial interpolation techniques – Radial Basis Functions (RBF) and Inverse Distance Weighting (IDW) were applied to the groundwater level data in three series of tests. Surface generation was obtained for each method. The water maps showed the spatial variation in the groundwater level in the study area and they are quite different. RBF method

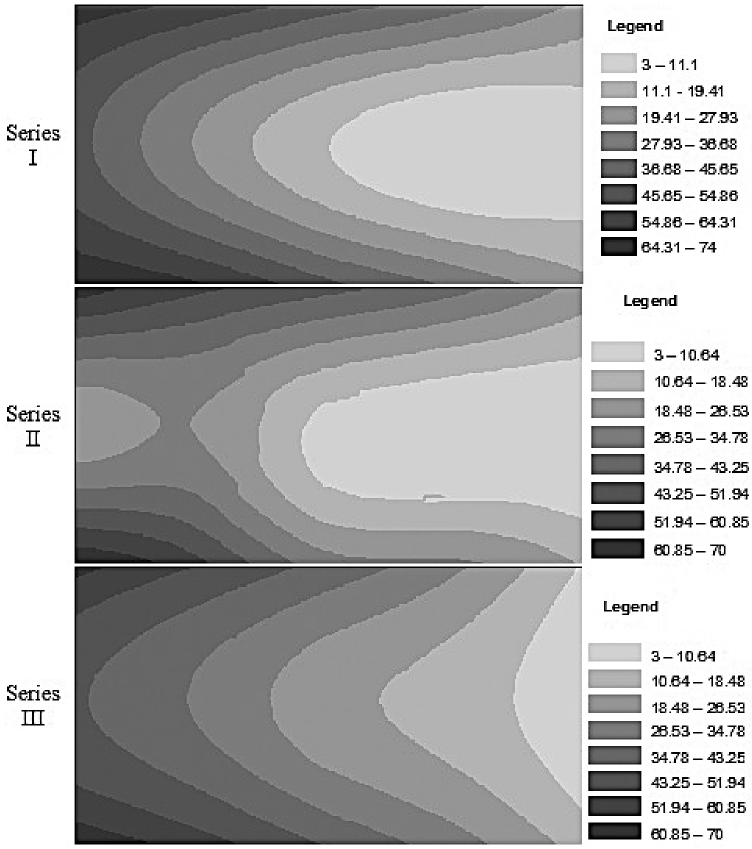

Figure 2. Prediction maps for RBF

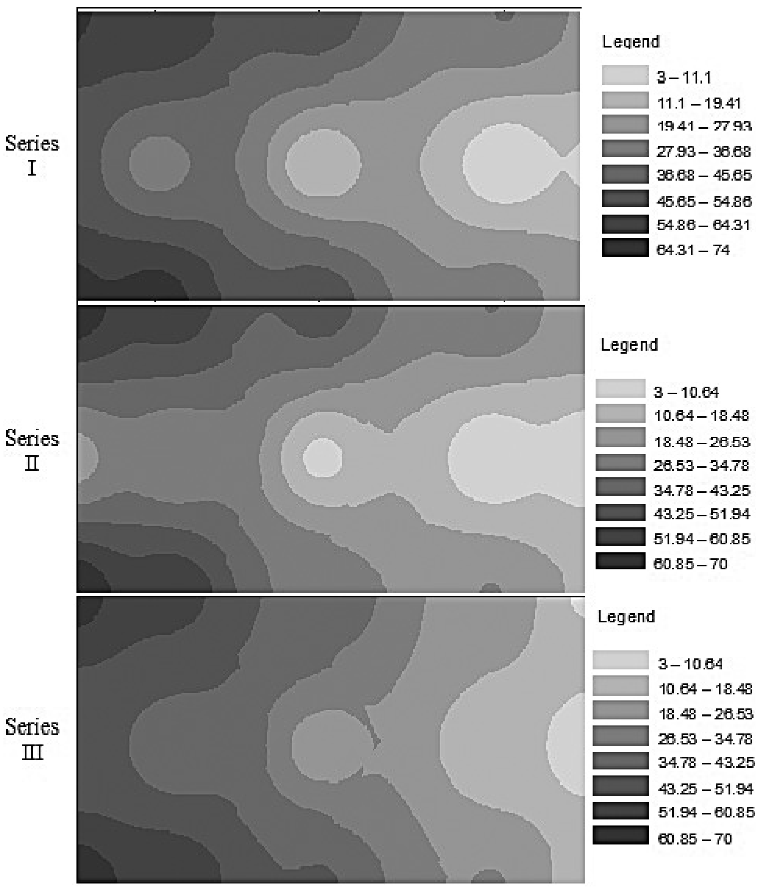

Figure 3. Prediction maps for IDW

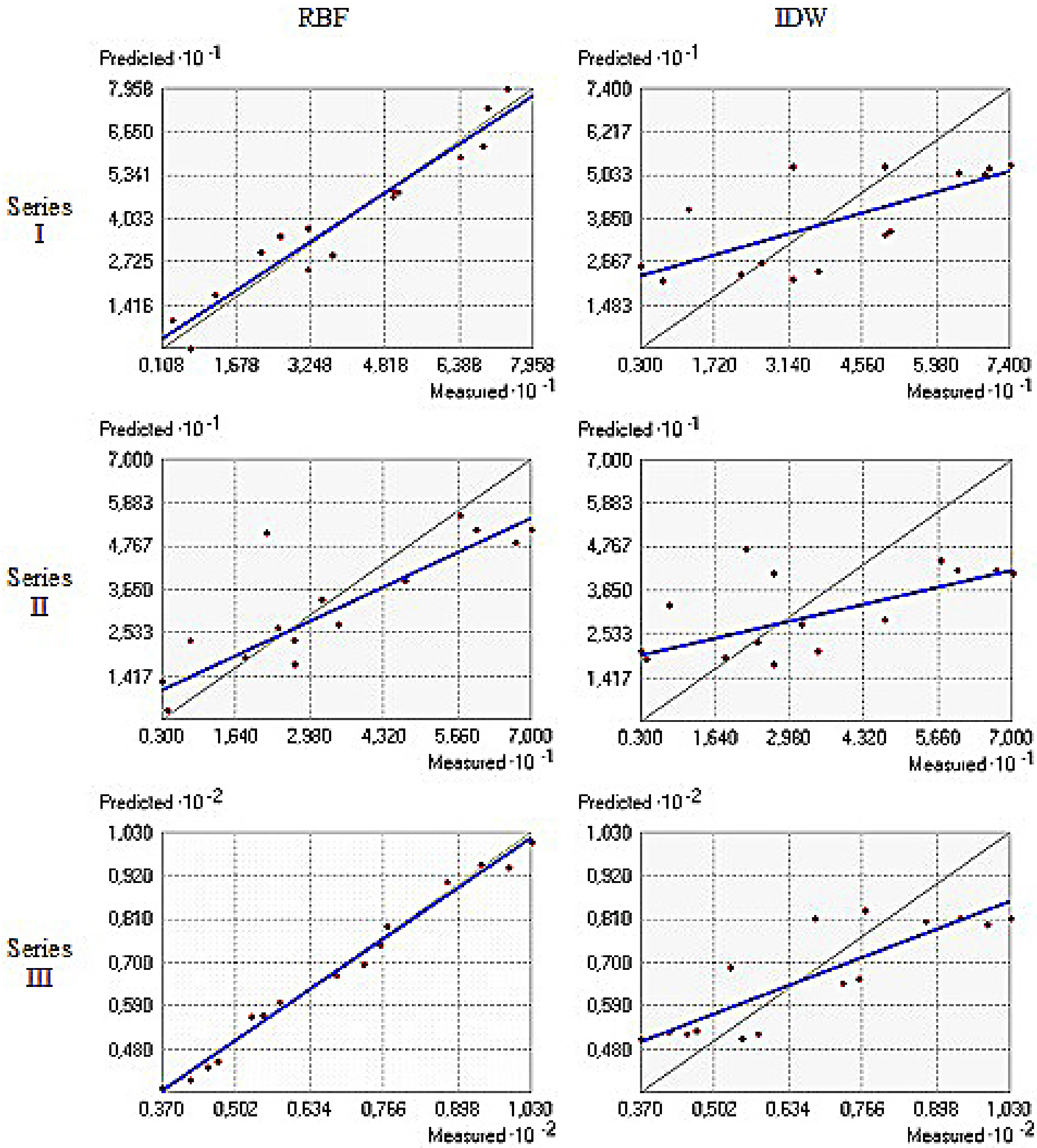

Figure 4. Scatter plots of interpolation methods of groundwater level data

has resulted in smoother map. The analysis of the methods of interpolation of analyzed data with the help of cross validation statistics and plots showed that Radial Basis Function creates better representation of reality for measured groundwater levels.

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