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Biogas production from chicken, sheep, and cow wastes in mono and co-digestion anaerobic pilot plants using modified Gompertz and first-order models: A case study in Erbil City

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ABSTRACT

Biogas production was investigated in pilot-scale digesters using chicken, sheep, and cow manure under mono- and co-digestion at ambient conditions in Erbil City, Kurdistan Region, Iraq, with a 40-day retention time. Mono-digestion trials applied five dilution ratios (1:1 to 1:3), while co-digestion used three different ratios (i.e. 1:1, 1:2, 1:3). Yield dynamics were analyzed through Modified Gompertz and First-order models, with statistical indicators (such as R^2, RMSE, RSS, AIC, and BIC) used to assess performance. Results demonstrated that chicken manure consistently achieved the highest cumulative biogas and CH_4 yield, reaching 323 L/kg VS and 194 L/kg VS at 1:1.5 under mesophilic regimes also, 320 L/kg VS and 192 L/kg VS at 1:2.5 ratios thermophilic regimes. While, sheep manure biogas yield peaked at 245 L/kg VS and CH_4 135 L/kg VS (ratio 1:1, and mesophilic), whereas cow manure biogas and CH_4 yield reached 279 L/kg VS and 153 L/kg VS (ratio 1:1.5, and mesophilic), showing improved adaptability under moderate dilution. Co-digestion revealed pronounced synergistic effects: at the 1:1 ratio under mesophilic conditions, cumulative yield reached 576 L/kg VS nearly double that of mono-digestion with smoother production curves and enhanced process stability. Kinetic modeling confirmed the superiority of the Modified Gompertz equation, which consistently provided robust fits (adjusted $R^2 > 0.95$, RMSE < 0.05, and low AIC/BIC), while the First-order model often produced unstable parameters. Regression analysis further validated the Gompertz model, presenting excellent agreement between predicted and experimental yields ($R^2 = 0.991$). This case study demonstrates the resilience of chicken manure, the moderate but dilution-sensitive performance of sheep manure, and the adaptability of cow manure under mesophilic regimes. Co-digestion provided clear synergistic benefits, and regression-based kinetic modeling was validated as a reliable framework for predicting biogas yield. Together, these findings deliver baseline insights for decentralized biogas production in the Kurdistan Region of Iraq.

Keywords: anaerobic pilot-plants, biogas yield, kinetic modeling, livestock manure, renewable energy.

INTRODUCTION

The increasing demand for sustainable energy solutions has intensified global interest in biogas production through anaerobic digestion, a biological process that converts organic matter into methane-rich gas under oxygen-free conditions. Biogas offers a renewable and decentralized energy source that simultaneously addresses waste management challenges and reduces greenhouse gas (GHG) emissions. Among the various feedstocks available, livestock manure is particularly attractive due to its high organic content, consistent availability, and potential for nutrient recycling (Nwuche et al., 2023).

Livestock manure from cattle, poultry, and sheep is rich in biodegradable volatile solids and trace elements that sustain microbial activity during anaerobic digestion. Biogas efficiency, however, depends strongly on operational factors such as substrate-to-water ratio and temperature, which influence substrate concentration, microbial accessibility, and microbial metabolism (Saleh and Hassan, 2021). Mono-digestion of a single feedstock often causes nutrient imbalances or inhibitory effects, whereas co-digestion enhances stability by improving C/N ratios, buffering capacity, and microbial diversity. Studies have shown that combining livestock manure with food waste, municipal organics, sewage sludge, or agricultural residues can increase methane yield by 25–400% compared to mono-digestion (Khanthong et al., 2023; Song et al., 2021). These findings highlight co-digestion as a practical strategy to overcome the limitations of single-substrate systems and maximize renewable energy recovery.

Despite these advancements, most existing research has focused on fixed mixing ratios and controlled laboratory temperatures, limiting the applicability of findings to real-world conditions. There is a notable gap in the literature regarding the systematic evaluation of how dynamic variations in substrate ratios and ambient temperature affect biogas production, particularly in a field-scale applications (Kaushal et al., 2024). Moreover, comparative studies involving different types of livestock manure such as cow, broiler chicken, and sheep under varying operational conditions remain scarce. This lack of comprehensive data hinders the development of adaptable and efficient an aerobic digestion system capable of responding to fluctuating environmental and feedstock characteristics (Chen et al., 2025; Khaliq et al., 2025).

For downstream applications, kinetic study of a bench-scale of an aerobic digestion process is crucial. Lag phase and hydrolysis constant are two ideal process variables that can be found by kinetic model fitting. Additionally, it assesses the activation energy of digestate pyrolysis and investigates the biogas generation rate and potential of an aerobic digestion process (Mishra and Mohanty, 2018). In addition to helping to forecast the biogas yield, hydraulic detention time, and energy consumption in the real application, the kinetic parameters derived from model fitting are essential for comprehending the reaction mechanism and optimizing the process parameters. The effects of binary and ternary trace elements an aerobic digestion were examined using both kinetic models and experimental measurements, which has rarely been reported by other studies, in light of the aforementioned benefits of kinetic model fitting (Wang et al., 2021). In order to ascertain the parameters of a biomethane potential test using nine organic substrates under thermophilic conditions, Nguyen et al. (2019) used four models. The Cone model outperformed the first-order and modified Gompertz models in terms of accuracy. Other investigations Li et al. (2015) showed that the Cone equation was more accurate, while other findings Santos et al. (2022) showed that the modified Gompertz and other models were appropriate. When launching a large-scale anaerobic digester, the batch study's findings, supported by the models' kinetic parameters, may offer some helpful foundations (Jijai and Siripatana, 2017). Several measurement techniques, such as R^2 (coefficient of determination), RMSE (root-mean-square error), AIC (Akaike information criterion), and BIC (Bayesian information criterion), were employed in this research to identify the best-fit equation. Important parameters might not be included in a single model, even though the kinetics might be used to assess the experimental data or determine the substrate degradability (Karki et al., 2021). For instance, the modified Gompertz and Logistic model does not evaluate the rate constant (k), therefore the lag time (λ) is not included in the Cone equation. In order to determine all significant parameters and their interpretation, it is necessary to consider the examination of several models and their consistency (Duong et al., 2025).

This study was conducted in Erbil City, the capital of the Kurdistan Region of Iraq, located in a semi-arid zone with hot summers exceeding 45 °C and cold winters below 0 °C (Mohamed et al., 2024). Such fluctuations directly affect microbial activity and digestion kinetics in anaerobic systems. Rapid urbanization has strained municipal infrastructure, particularly wastewater and solid waste management(Mohammad et al., 2025). Livestock farming in the surrounding governorate generates large amounts of chicken, sheep, and cow manure, which are often unmanaged and contribute to pollution and greenhouse gas emissions (Saleh and Hassan, 2021). These abundant organic wastes provide a relevant substrate mix for anaerobic digestion research in Erbil (Ismael and Aziz, 2026). The work was carried out in pilot-scale batch digesters operated under ambient conditions unlike laboratory studies conducted under constant conditions testing different substrate-to-water dilution ratios. Physicochemical characteristics and gas composition were contextualized using literature values, while the main focus was on yield dynamics, kinetic modeling, and co-digestion synergy. This case study provides baseline data for future continuous-flow designs applicable to semi-arid urban environments.

The objective of this study was to provide one of the first systematic pilot-scale evaluations of mixed livestock manures under ambient conditions in Erbil, Iraq. By examining chicken, sheep, and cow manure in both mono- and co-digestion configurations, the research aimed to assess cumulative biogas yield, process stability, and the synergistic benefits of co-digestion. A further goal was to rigorously apply and compare the Modified Gompertz and First-order kinetic models, supported by statistical indicators and regression validation, to establish a reproducible framework for predicting biogas dynamics. Through this approach, the study sought to generate baseline data that can guide the optimization of anaerobic digestion systems and inform future field-scale applications in semi-arid urban environments.

METHODOLOGY

Substrate collection and preparation

Fresh chicken, sheep, and cow manure were obtained from local farms in Erbil City- Iraq and used as feedstock for anaerobic digestion experiments. Prior to use, visible impurities such as bedding materials and stones were removed to ensure uniformity. For mono-digestion trials, the substrates were diluted with tap water to achieve five substrate-to-water (S/W) ratios: 1:1, 1:1.5, 1:2, 1:2.5, and 1:3. Co-digestion trials were prepared by homogenizing equal proportions of chicken, sheep, and cow manure, which were subsequently diluted at three S/W ratios: 1:1, 1:2, and 1:3. The initial pH of all mixtures was adjusted to approximately 7.3 to maintain favorable conditions for microbial activity. Tap water was utilized to maintain standardized conditions and minimize biochemical interference during the kinetic model validation phase. Thorough mixing was performed before loading to ensure uniform distribution of substrates and consistent microbial inoculation.

Pilot-scale experimental setup and operational parameters

Anaerobic digestion experiments were performed in a pilot-scale facility designed to replicate decentralized rural energy systems. Three identical reactors, each with a working volume of 60 liters, were fabricated from high-density polyethylene (HDPE) barrels. The digesters were fitted with airtight lids, gas outlet tubing, biogas flow meters (m^3), pressure gauges (mbar), and digital sensors for continuous monitoring of ambient temperature and humidity. The system was operated in batch mode under controlled conditions, with a fixed HRT of 40 days to allow for complete degradation of the biodegradable fraction as shown in (Figure 1).

Ambient temperature was recorded daily, ranging between 18 °C and 50 °C, thereby encompassing psychrophilic (18–24 °C), mesophilic (30–40 °C), and thermophilic (40–50 °C) regimes (Humphrey et al., 2025). Internal headspace pressure was monitored using analog manometers, while biogas accumulation was directed into expandable low-pressure reservoirs to

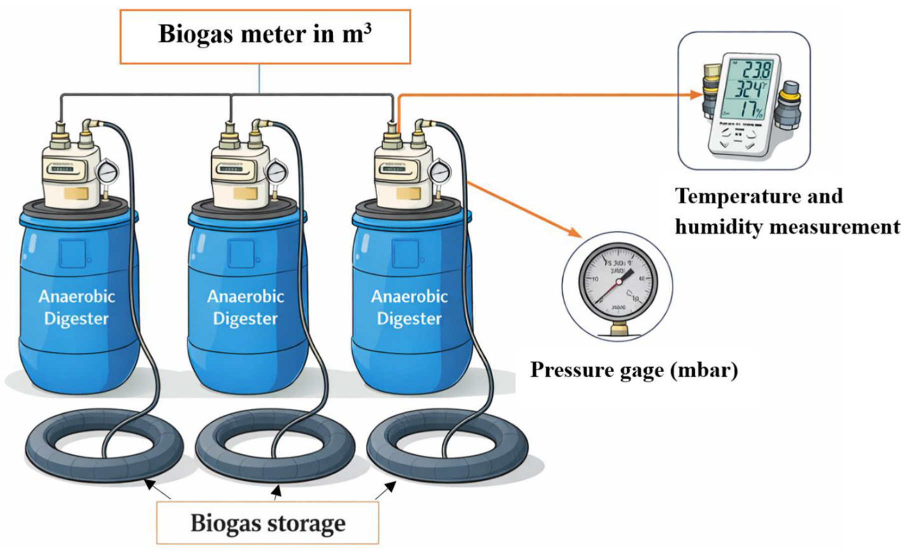

Figure 1. Pilot-scale anaerobic digestion system with digesters, biogas meters (m^3), pressure gauge (mbar), and temperature/humidity sensor connected to biogas storage units

prevent back-pressure interference with gas flow measurements. Biogas production was quantified as daily flow rate in cubic meters (m^3), providing the primary performance indicator for all experimental conditions.

Experimental design

Mono-digestion experiments were carried out individually for chicken, sheep, and cow manure across five substrate-to-water (S/W) ratios: 1:1, 1:1.5, 1:2, 1:2.5, and 1:3. The digesters were operated under ambient temperature conditions ranging between 18 °C and 50 °C. At the 1:1 ratio, 24 kg of animal waste were combined with 24 liters of water, corresponding to equal proportions of substrate and dilution. For the 1:1.5 ratio, 19 kg of manure were mixed with 29 L of water. The 1:2 ratio involved 16 kg of manure diluted with 32 liters of water, equivalent to one-third substrate and two-thirds water. At the 1:2.5 ratio, 14 kg of manure were blended with 34 L of water. Finally, the 1:3 ratio consisted of 12 kg of manure mixed with 36 L of water, representing one-quarter substrate and three-quarters water.

Following the completion of mono-digestion trials, co-digestion experiments were conducted under mesophilic conditions (30–40 °C) between 10 September and 1 November 2025. Equal proportions of chicken, sheep, and cow manure were homogenized to form composite feedstock, which was subsequently diluted at three S/W ratios. In the first configuration (1:1), 24 kg of composite waste (8 kg from each source) were mixed with 24 L of water. The second configuration (1:2) involved 16 kg of composite waste (approximately 5.5 kg from each source) combined with 32 L of water. The final configuration (1:3) utilized 12 kg of composite waste (4 kilograms from each source) diluted with 36 L of water. Each mixture was thoroughly blended prior to loading to ensure uniform distribution of substrates and consistent microbial activity throughout the digestion process.

Characterization of animal manure

Animal waste serves as a renewable and cost-effective substrate for biogas production via anaerobic digestion. If improperly handled, it can threaten the environment due to high nitrogen, phosphorus, pathogens, and heavy metals. However, using anaerobic digestion technology converts this waste into sustainable biogas, while ensuring proper waste management. Viable animal waste sources include manure from chicken, sheep, and cow noted for their nutrient content and organic matter concentration. Variations in manure's characteristics arise from factors such as animal type, diet, and management systems. Key parameters for assessing methane production potential in animal waste include volatile solids, total solids, pH, and C/N ratio, as shown in (Table 1).

Chicken manure is known for its high nitrogen content and a low C/N ratio of 7.5 to 9.7, making it highly degradable but susceptible to ammonia inhibition at elevated organic loading. TS can range from 20% to 92%, while VS are between 18% and 84%, influenced by diet and management. Methane yields vary from 157 to 395 L/kg VS, with higher yields in controlled conditions. Although it has a high buffering capacity that supports microbial activities, excessive TS can limit water availability, hindering nutrient transport (Shen et al., 2019). In addition, sheep manure has intermediate characteristics, with TS values of 22% to 40% and VS between 19% and 73%. Its C/N ratio is typically between 11 and 15, reducing ammonia inhibition risk while providing adequate nitrogen for microbial activities. CH_4 yields range from 160 to 396 L/kg VS, indicating moderate degradability. Composition can vary based on seasonal diet and bedding materials (Achinas et al., 2018).

Table 1. Physiochemical characteristics and methane yield of (chicken, sheep, and cow) manure

Characteristics of animal waste	Chicken manure	Sheep manure	Cow manure
pH	6.9–7.4	7.16–8.1	7.1–8.6
TS (%)	20–92.6	22.3–40	14.5–22.7
VS (%)	18.3–84.1	18.7–72.7	11.9–72
Moisture content (%)	66.0–71.5	71.0–76.0	80.5–84.0
C/N ratio	7.5–9.75	11.3–14.7	14.59–18.9
CH_4 yield (L/kg VS)	157.0–395.0	160.0–396.0	207.0–357.0
References	(Shen et al., 2019)	(Achinas et al., 2018)	(Li et al., 2020)

Additionally, cow manure features lower TS (14% to 23%) and higher moisture (80% to 84%), which dilutes its organic content. Its VS content varies from 12% to 72%, but high lignin content from feed reduces biodegradability. The CH_4 yields for cow manure range from 207 to 357 L/kg VS, generally lower than sheep or poultry. However, its higher C/N ratio (15 to 20) aids in stable digestion and minimizes ammonia inhibition, making it a beneficial co-substrate when mixed with nitrogen-rich manures (Li et al., 2020).

Statistical analysis for kinetic modeling

Two mathematical models were employed to describe the cumulative biogas production observed during the pilot-scale anaerobic digestion trials: the Modified Gompertz equation and the First-order kinetic model. The Modified Gompertz equation (Equation 1) was employed as one of the primary kinetic models to estimate biogas production dynamics during anaerobic digestion. This model incorporates lag time (λ), representing the initial period before measurable gas generation begins, as a key variable. Originally developed for predicting human mortality rates, the Gompertz function has since been adapted extensively for microbial growth-related processes, including anaerobic digestion (Navarro et al., 2022). The fundamental assumption underlying the model is that microbial activity follows a sigmoidal growth pattern, where the rate of gas production increases exponentially after the lag phase and subsequently stabilizes as substrate availability declines (Dupont et al., 2023).

$$B_t = B_{max.}\ exp \{-exp [\frac{R_{max}.e}{B_{max}} (\lambda - t) + 1]\} s \quad (1)$$

Cumulative biogas yield B_t can be described using three kinetic approaches. In the Modified Gompertz model, B_{max} represents the maximum biogas potential (L/kg VS), R_{max} is the peak production rate (L/kg VS. day), λ denotes the lag phase (days), and e is Euler's constant (2.71828).

The First-order model, originally derived from microbial growth kinetics (Equation 2), has been widely applied to estimate the maximum biogas potential and the hydrolysis rate constant (k, day^{-1}) (Wang et al., 2021). This approach assumes a constant reaction rate and a single-substrate system without intermediate steps, making it suitable for anaerobic digestion processes involving animal manures and other organic feedstocks. Its simplicity and reliability have led to its frequent use in predicting degradation dynamics under batch digestion conditions (Dupont et al., 2023).

$$B_t = B_{max.} (1 - e^{-kt}) \quad (2)$$

First-order kinetic model relates B_{max} (L/kg VS) to a degradation rate constant k (day), assuming gas production is directly proportional to the remaining biodegradable fraction and providing a simple yet effective description of first-order dominated systems.

Goodness of fit for the chosen models can be quickly judged by adjusted-R^2 (Equation 3) values. However, there are several other criteria that can determine the best model for predicting an experimental data set (Pererva et al., 2020). This study also included the residual sum of squares (RSS, Equation 4) also known as the sum of squared estimate of errors (SSE), and root mean square error (RMSE, Equation 5) for goodness of fit (Şenol, 2023). This study also calculated the corrected Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) which are two criteria used to reduce overfitting in the model, where lower values indicate a better fitting to the model (Li et al., 2018). AIC and BIC formulas are detailed in Equation 6 and Equation 7, respectively:

$$Adjusted\ R^2 = 1 - \frac{(1-R^2)(n-1)}{n-p-1} \quad (3)$$

$$RSS = SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (4)$$

$$RMSE = \sqrt{\frac{1}{n} RSS} \quad (5)$$

$$AIC = n \times ln \left(\frac{RSS}{n}\right) + 2K + \frac{2K(K+1)}{n-K-1} \quad (6)$$

$$BIC = ln \left(\frac{RSS}{n}\right) + K + ln(n) \quad (7)$$

where: yi is the experimental value, yi is the predicted value, y is the mean value, n is the number of data points, p is the number of independent variables, RSS is the residual sum of squares, and K is the number of parameters the model is used to fit plus one (Li et al., 2018).

All experimental data were analyzed using OriginPro 2021 software. Daily biogas production was compiled as cumulative yields based on substrate-to-water ratios and digestion conditions. Nonlinear regression functions fitted the experimental data to the Modified Gompertz and

First order kinetic models, allowing estimation of model parameters such as lag phase duration, maximum production rate, cumulative yield, and rate constants from the regression outputs.

RESULTS AND DISCUSSION

Biogas production in mono-digestion

The mono-digestion experiments revealed clear differences in biogas production depending on substrate type, dilution ratio, and operating temperature as shown in (Figure 2). Chicken manure consistently demonstrated the fastest onset of gas generation, with yields exceeding 13 L/kg VS.day by the third day under mesophilic conditions at the 1:1 ratio. Sheep manure followed with peak values around 10–11 L/kg VS.day, while cow manure showed negligible production during the initial days and only modest yields thereafter. This pattern indicates that chicken and sheep manure contained more readily degradable organic fractions, whereas cow manure required longer microbial adaptation before measurable

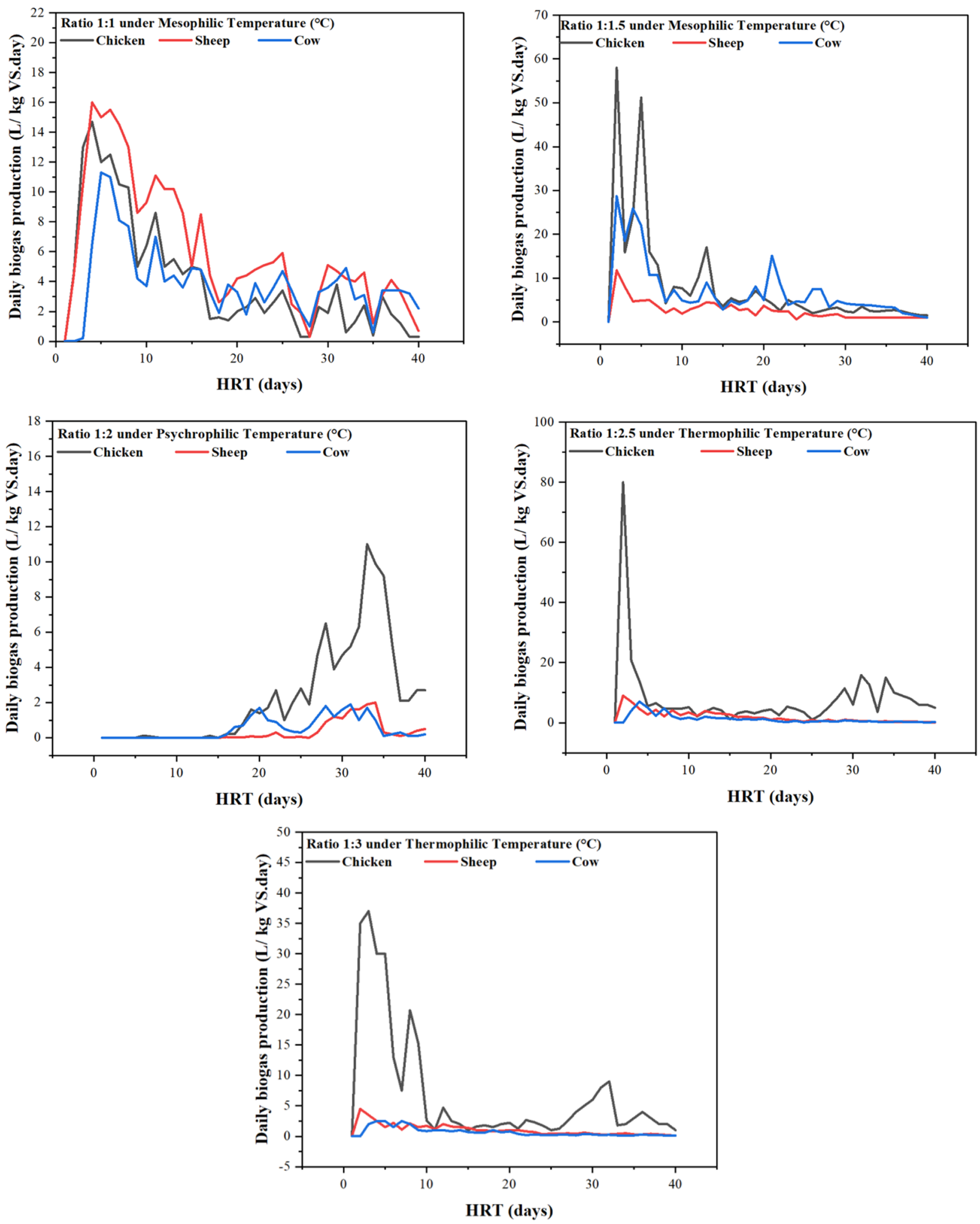

Figure 2. Daily biogas production record during digestion process in mono-digester system with various ratios

gas release occurred (Morvan and Nicolardot, 2009). Additionally, at the 1:1.5 ratio under mesophilic conditions, chicken manure achieved exceptionally high yields, peaking at 58 L/kg VS.day on day 2 before stabilizing around 15–25 L/kg VS.day. Sheep manure reached 11.8 L/kg VS.day, while cow manure achieved 28.7 L/kg VS.day, suggesting that moderate dilution enhanced cow manure performance compared to the 1:1 condition. Psychrophilic trials at the 1:2 ratio showed slower overall gas evolution, with chicken manure peaking at 16 L/kg VS.day, sheep manure at 4.7 L/kg VS.day, and cow manure at 25.9 L/kg VS.day. Interestingly, cow manure performed relatively better under psychrophilic conditions, highlighting its microbial adaptability at lower temperatures when sufficient dilution was provided (Massé and Cata Saady, 2015). However, thermophilic trials at the 1:2.5 ratio revealed a striking dynamic. Chicken manure exhibited a pronounced surge of 80 L/kg VS.day on day 2, representing the highest single-day yield observed across all experimental conditions. This peak was followed by a sharp decline to 20.6 L/kg VS.day on day 3, after which production stabilized at lower levels. Sheep manure peaked at 7 L/kg VS.day, while cow manure remained below 5 L/kg VS.day. At the 1:3 ratio under thermophilic conditions, chicken manure produced 35 L/kg VS.day by day 2, while sheep manure remained below 5 L/kg VS.day and cow manure was negligible. These results confirm that chicken manure maintained strong biogas potential even under high dilution and elevated temperatures, whereas sheep and cow manure were more sensitive to thermophilic regimes (Al-Zoubi et al., 2024).

The influence of dilution on gas yield was evident across all substrates. At low dilution (1:1), chicken and sheep manure produced high initial yields but experienced rapid declines, suggesting substrate inhibition or accumulation of volatile fatty acids (Al-Zoubi et al., 2024). Cow manure performed poorly at this ratio, likely due to its higher fiber content and slower hydrolysis rate (Karami-Boozhani et al., 2024). At moderate dilution (1:1.5 and 1:2), cow manure exhibited improved yields, particularly under psychrophilic conditions, where cumulative production surpassed sheep manure. Chicken manure also benefited from moderate dilution, showing smoother production curves and reduced fluctuations compared to the 1:1 ratio (Mohammed et al., 2023). At higher dilution (1:2.5 and 1:3), chicken manure maintained strong yields, particularly under thermophilic conditions, where daily production exceeded 30 L/kg VS.day. Sheep manure showed moderate improvements, while cow manure remained inconsistent, with negligible yields at 1:3 despite improved performance at 1:2. Excessive dilution likely reduced substrate concentration below optimal levels, limiting microbial activity and gas generation. These findings align with previous studies on pig and dairy manure digestion, which reported that moderate dilution enhances hydrolysis and microbial access, while excessive dilution reduces substrate concentration below optimal levels, thereby limiting microbial activity (Wang et al., 2023).

Temperature also exerted a strong influence on biogas production dynamics. Under mesophilic conditions (30–40 °C), chicken manure consistently achieved the highest yields, with rapid onset and sustained production, which agrees with recent findings that mesophilic regimes provide stable CH_4 yields in livestock manure digestion (Steiniger et al., 2023). Sheep manure performed moderately well, while cow manure lagged behind except at the 1:1.5 ratio, where dilution improved its performance. Psychrophilic conditions (18–24 °C) slowed overall gas evolution, but cow manure showed relatively better adaptation compared to sheep manure, consistent with reports that psychrophilic microbial consortia possess broader metabolic flexibility. Chicken manure maintained reasonable yields under psychrophilic regimes, though peak values were lower than those observed under mesophilic and thermophilic conditions (Alblooshi et al., 2025). Thermophilic conditions (40–50 °C) produced mixed results. Chicken manure exhibited strong yields at both 1:2.5 and 1:3 ratios, suggesting that its substrate composition supported rapid microbial activity at elevated temperatures, in line with studies showing poultry manure responds strongly to thermophilic methanogenesis (Makhsumkhanov et al., 2023). Sheep manure showed moderate yields, while cow manure was highly sensitive, with negligible production at 1:3. The reduced performance of cow manure under thermophilic conditions may be linked to the inhibitory effects of high temperature on fiber-degrading microbial populations, a trend also noted in cattle manure digestion studies. These observations are consistent with reports that mesophilic regimes generally provide the most stable yields, while thermophilic conditions favor substrates rich in easily degradable organic matter (Steiniger et al., 2023).

Comparative evaluation revealed that chicken manure was the most effective feedstock for biogas production across all experimental conditions. Its rapid onset of gas generation, high peak yields, and resilience to dilution and temperature variations suggest that it contains readily degradable organic matter, such as proteins and lipids, which support fast microbial metabolism (Karami-Boozhani et al., 2024). Sheep manure produced moderate yields, with performance strongly influenced by dilution and temperature. Its intermediate degradability likely reflects a balance between easily hydrolyzed fractions and more recalcitrant components (Mohammed et al., 2023). Cow manure exhibited the most variable performance, with poor yields at low dilution and under thermophilic conditions, but improved production under psychrophilic regimes with moderate dilution. This variability can be attributed to its higher lignocellulosic content, which slows hydrolysis and requires longer microbial adaptation (Rollo et al., 2017). The improved performance under psychrophilic conditions suggests that cow manure may harbor microbial consortia capable of functioning effectively at lower temperatures, though its overall biogas potential remains lower than chicken manure (Karami-Boozhani et al., 2024; Mohammed et al., 2023).

Comparative analysis showed that chicken manure proved the most effective feedstock, with rapid gas onset, high yields, and strong tolerance to dilution and temperature, reflecting its readily degradable organic content that supports fast microbial metabolism (Karami-Boozhani et al., 2024). Sheep manure produced moderate yields, strongly influenced by temperature and dilution, consistent with its intermediate degradability between easily hydrolyzed and recalcitrant fractions (Mohammed et al., 2023). Cow manure showed the most variable performance, with poor yields under thermophilic conditions but improved output at lower temperatures, attributable to its higher lignocellulosic content and slower hydrolysis. This variability suggests the presence of microbial consortia adapted to psychrophilic regimes, though overall biogas potential remains lower than chicken manure (Li et al., 2022; Telo and Yigezu, 2025).

Biogas production in co-digestion

The co-digestion trials revealed distinct differences in gas production depending on substrate ratio, HRT, and operating temperature. At the early stages (days 1–3), no measurable gas was detected across all ratios, reflecting the lag phase of microbial adaptation as illustrated in (Figure 3). By day 4, biogas production commenced, with the 1:1 ratio yielding 12 L/kg VS.day, while the 1:2 and

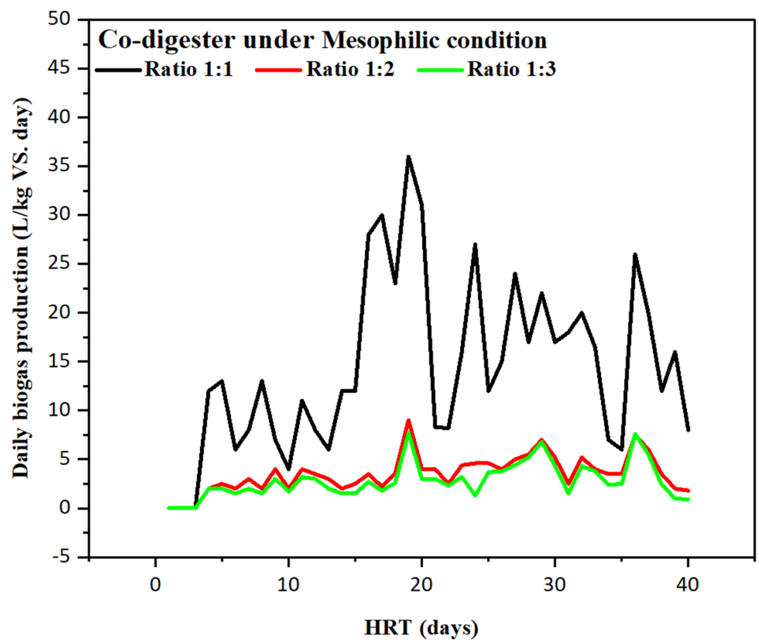

Figure 3. Anaerobic co- digester mixing of (chicken, sheep, and cow manure) daily biogas production in (L/kg VS.day)

1:3 ratios produced only 2–2.5 L/kg VS.day. This early onset at the 1:1 ratio indicates that higher substrate concentration accelerated microbial activity once acclimation was achieved. Peak yields were observed between days 16 and 20. At day 16, the 1:1 ratio reached 28 L/kg VS.day, while the 1:2 and 1:3 ratios produced 3.5 and 2.7 L/kg VS.day, respectively. By day 19, the 1:1 ratio achieved its maximum of 36 L/kg VS.day, compared to 9 L/kg VS.day at 1:2 and 7.7 L/kg VS.day at 1:3. This demonstrates that co-digestion at higher substrate concentration consistently outperformed diluted systems, both in terms of peak yield and cumulative production. Temperature variations between 26–36 °C influenced performance. The highest yields coincided with temperatures around 35–36 °C, confirming that mesophilic conditions favored stable microbial activity. At lower temperatures (26–28 °C), yields dropped significantly, with the 1:1 ratio falling to 6–12 L/kg VS.day and the diluted ratios producing only 2–4 L/kg VS.day. This pattern highlights the sensitivity of co-digestion systems to temperature shifts, with mesophilic ranges providing optimal stability. The effect of dilution was evident across all ratios. At 1:1, gas production was consistently higher, with smoother curves and stronger peaks. At 1:2, yields were moderate, peaking at 9 L/kg VS.day, while at 1:3, yields remained low, rarely exceeding 7.7 L/kg VS.day. Excessive dilution reduced substrate concentration below optimal levels, limiting microbial activity and gas generation. These findings align with reports on pig and dairy manure digestion, where moderate dilution enhanced hydrolysis and microbial access, but excessive dilution reduced productivity (Wang et al., 2023).

Composition of biogas yield

Temperature exerted a consistent influence on CH_4 composition in both mono- and co-digestion systems. In mono-digestion, chicken manure showed the highest CH_4 fractions, rising from 45–55% under psychrophilic conditions to 55–65% in mesophilic ranges and 60–68% in thermophilic regimes (Alblooshi et al., 2025; Steiniger et al., 2023) while sheep manure progressed more modestly from 45–52% to 50–58% and 55–62%, and cow manure increased from 45–50% to 55–60% and 58–65% across the same temperature ranges (Karami-Boozhani et al., 2024; Massé and Cata Saady, 2015). Co-digestion of chicken, sheep, and cow manure followed a similar temperature-dependent trend, with methane content modest at 50–56% under psychrophilic ranges, increasing to 59–63% under mesophilic conditions, and reaching 60–68% in thermophilic operation (Almomani and Bhosale, 2020). Taken together, these findings demonstrate that CH_4 percentage consistently rises with temperature, with mesophilic conditions providing the most stable performance, while co-digestion further enhances gas quality compared to individual substrates. The CH_4 yields at HRT 40 days from mono-digestion of chicken, sheep, and cow manure and co-digestion of mixed livestock manure as shown in (Figure 4).

At an HRT of 40 days, CH_4 yields exhibited clear distinctions between mono- and co-digestion systems. In the mono-digestion experiments, chicken manure consistently produced the highest CH_4 output, with peak values exceeding 190 L/kg VS, reflecting its readily degradable protein and lipid fractions. Sheep manure displayed moderate yields, ranging from ~7 to 135 L/kg

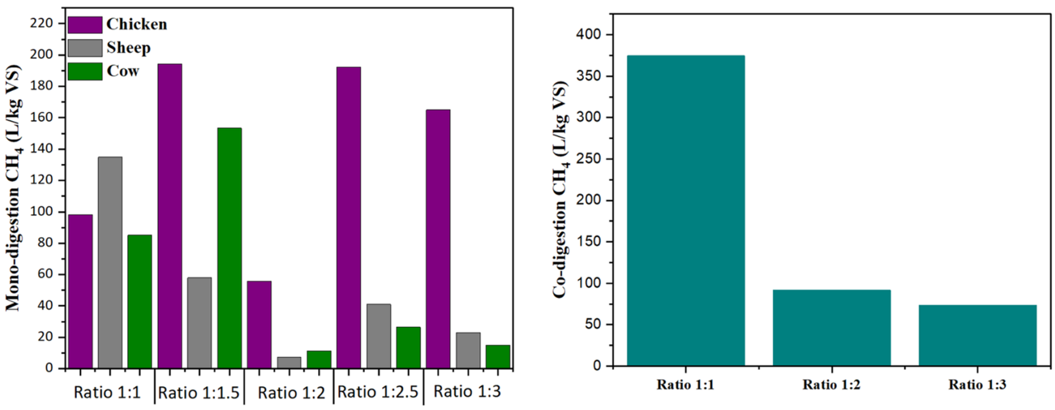

Figure 4. CH_4 yields at HRT 40 days from mono-digestion of chicken, sheep, and cow manure compared with co-digestion of mixed livestock manure

VS, constrained by its fibrous composition and slower hydrolysis, while cow manure produced comparatively lower values (11–153 L/kg VS), consistent with its lignocellulosic nature. These results highlight the substrate-specific limitations of single-feedstock digestion, where nutrient imbalances and inhibitory effects reduce methane productivity over extended retention times.

In contrast, the co-digestion system at HRT 40 days demonstrated a pronounced synergistic enhancement. CH_4 yield reached ~374 L/kg VS under mesophilic operation, nearly double the highest mono-digestion value, confirming the benefits of balanced substrate composition. This improvement is attributed to the optimized carbon-to-nitrogen ratio achieved by mixing nitrogen-rich chicken manure with the more carbonaceous sheep and cow manures, which mitigates ammonia inhibition and sustains methanogenic activity. Additional mechanisms include enhanced microbial diversity, improved buffering capacity, and cross-stimulation of hydrolytic and acetogenic communities. Although CH_4 yield declined under more diluted conditions (~91–73 L/kg VS), co-digestion consistently outperformed mono-digestion, demonstrating robustness and scalability under the 40-day retention period.

Collectively, these findings establish that CH_4 fraction and yield consistently rise with temperature and balanced substrate composition, with mesophilic co-digestion at HRT 40 days providing the most stable and reproducible enhancement. This confirms its practical potential for integrated waste management and renewable energy generation in regions with naturally occurring mesophilic climates.

The decisive influence of substrate type, dilution ratio, and operating conditions on biogas yield has been consistently highlighted in previous investigations. Under mesophilic conditions, cow dung was reported by Arekemase and Aweda (2021) to produce the highest output, while co-digestion with chicken droppings and rice husk resulted in only moderate improvements, indicating the importance of balanced nutrient composition. Enhanced methane yield and process stability were observed by Chen et al. (2023) when cow and sheep manure were co-digested at 25 °C, confirming the critical role of mesophilic ranges in sustaining microbial activity. The sensitivity of poultry manure to temperature was demonstrated by, Al-Zoubi et al. (2024) with maximum methane production achieved at 37 °C, underscoring temperature as a decisive parameter. Similarly, Saadia et al. (2025) identified cow dung as the most efficient substrate compared to pig dung and vegetable waste, achieving 0.391 m³ of biogas with 69% CH_4 content under mesophilic conditions.

Kinetics of the anaerobic digestion

The kinetic modeling of biogas production from chicken, sheep, and cow manures under co-digestion and mono-digestion conditions was evaluated using both the Modified Gompertz and First-order models as shown in (Figure 5 and 6).

The fitted parameters included B_{max}, R_{max}, λ, k, and adjusted R^2 as shown in (Table 2). The Modified Gompertz model provided robust fits for most mono-digester and co-digester datasets, with adjusted R^2 values consistently above 0.95. For chicken manure mono-digestion, the B_{max} ranged from 145 L/kg VS (ratio 1:2) to 310 L/

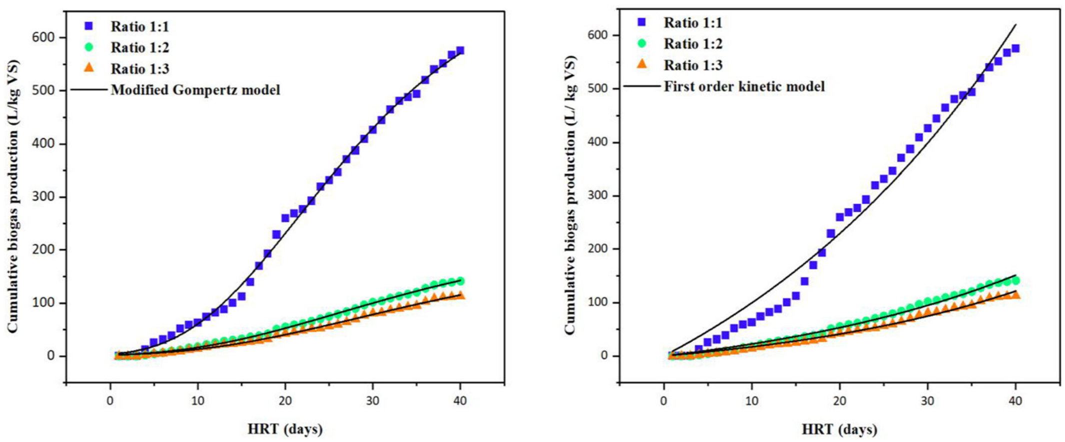

Figure 5. Kinetic modeling results for co-digestion system mixing (chicken, sheep, and cow manure) using, modified Gompertz and first-order equation

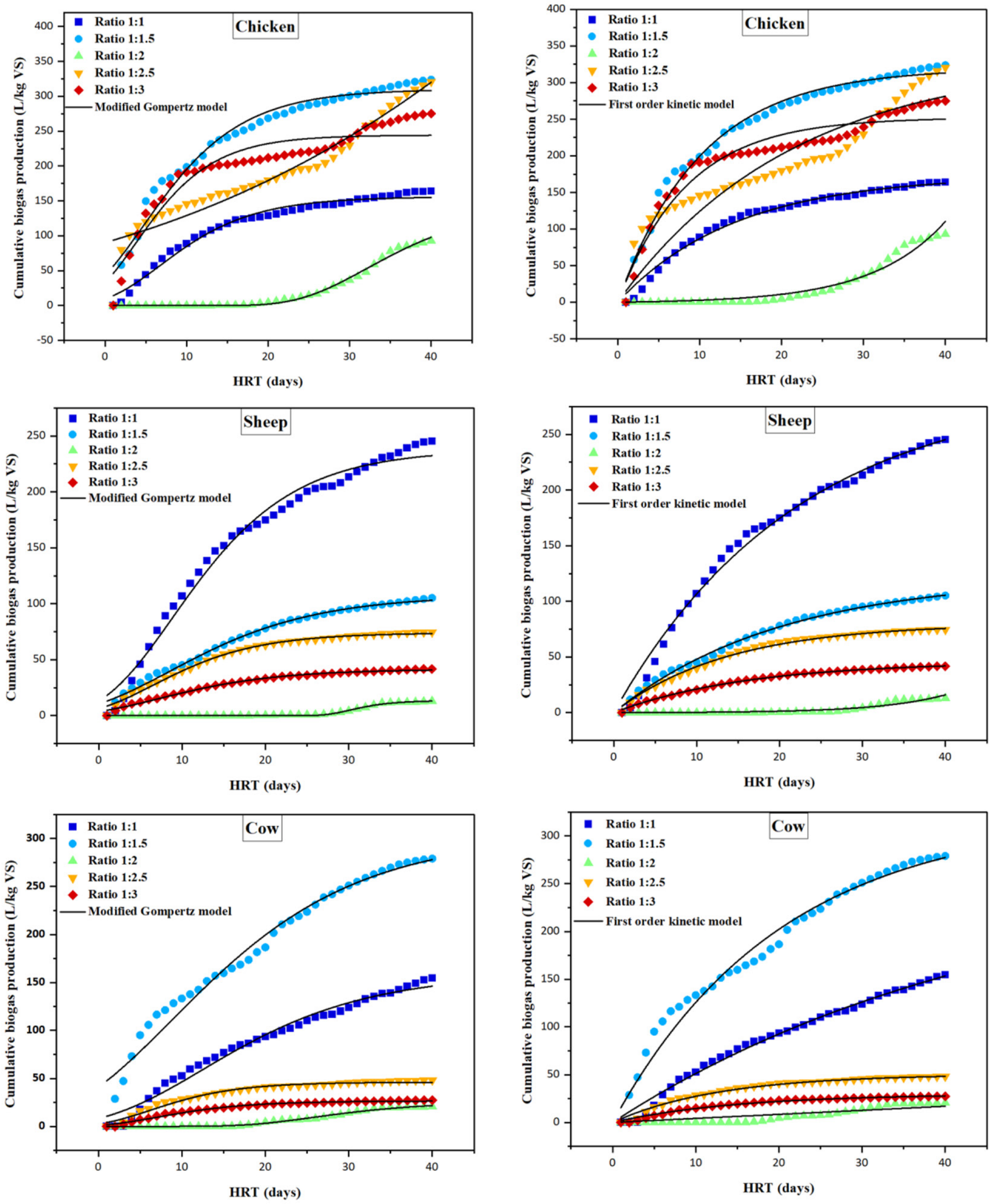

Figure 6. Kinetic modeling results for mono-digestion system under different substrate and mixing ratio using, modified Gompertz and first-order equation

kg VS (ratio 1:1.5). The maximum production rate R_{max} was highest at ratio 1:1.5 (16.55 L/kg VS.day), while the λ shortened to 4.68 days, indicating rapid microbial adaptation. Deviations between experimental and predicted yields at day 40 were modest (Δ = 4–5%), confirming the reliability of the model. However, the extreme B_{max} value at ratio 1:2.5 (7648 L/kg VS) reflects over-parameterization and should be interpreted cautiously. In addition, for sheep manure mono-digestion, Gompertz modeling produced realistic yields across all ratios. At ratio 1:1, B_{max} was 238 L/kg VS with R_{max} 10.5 L/kg VS.day and Δ = 5.2%. Ratios 1:2 and 1:2.5 yielded smaller volumes (13–74 L/kg VS) but with excellent predictive accuracy (Δ < 2%). Cow manure showed similar consistency, with ratio 1:1.5 producing 303 L/kg VS and Δ = 0.36%, while ratio 1:2 yielded 25.6 L/kg VS with Δ = 5.5%. Overall, Gompertz modeling captured the kinetics of mono-digestion effectively, though occasional anomalies highlight the need for careful parameter validation (Gomes et al., 2021). However, in co-digestion systems, Gompertz modeling

Table 2. Summary of the kinetic parameters from the models for anaerobic mono and co- digestion

Modified Gompertz model								
Substrate	Ratio	$B_{(max)}$ (L/kg VS)	R_{max} (L/kg VS. day)	λ (days)	Biogas yield at day 40 (L/kg VS)		Δ (%)	Adjusted R^2
					Experimental	Predicted		
Mono-digester Chicken	1:1	155.7 ± 2.099	8.73	6.75	163.8	154.8	5.5	0.97967
	1:1.5	310.04 ± 4.811	16.55	4.68	323.8	308.2	4.8	0.96355
	1:2	145.32 ± 11.93	6.35	32.16	93	97	4.3	0.99347
	1:2.5	7648.5±30190	23.6	177.62	320.7	320.12	0.18	0.92923
	1:3	244.27 ± 5.136	16.5	3.79	275	243.95	11.3	0.89716
Mono-digester Sheep	1:1	237.96 ± 4.15	10.5	8.87	245.2	232.41	5.2	0.98362
	1:1.5	107.64 ± 1.6	3.95	8.53	105.2	103.06	2	0.9914
	1:2	13.4365 ± 0.3	1.69	30.16	13.03	12.978	0.4	0.99412
	1:2.5	73.96 ± 0.61	3.78	6.54	74.5	73.27	1.65	0.99267
	1:3	41.399 ± 0.43	1.86	7.28	41.65	40.64	2.4	0.99207
Mono-digester Cow	1:1	159.79 ± 5.72	5.14	12.43	154.7	146.1	5.6	0.97911
	1:1.5	303.48 ± 11.12	8.74	9.91	279	278	0.36	0.97213
	1:2	25.6 ± 1.292	1.18	25.77	20.55	21.67	5.5	0.99048
	1:2.5	46.095± 0.594	2.86	6.28	48	46	4.2	0.97715
	1:3	26.418 ± 0.293	1.56	7.14	27.4	26.3	4	0.98618
Co-digester	1:1	728.97 ± 20.3	20.7	21.77	576	571.1133	0.85	0.99733
	1:2	221.12 ± 8.13	4.82	26.09	141	142.6393	1.16	0.99815
	1:3	195.87 ± 10.84	3.93	28.36	113	115.255	2	0.99706

First order kinetic model							
Substrate	Ratio	$B_{(max)}$ (L/kg VS)	k (day^{-1})	Biogas Yield at Day 40 (L/kg VS)		Δ (%)	Adjusted R^2
				Experimental	Predicted		
Mono-digester Chicken	1:1	173.29 ± 3.018	0.06947 ± 0.003	163.8	162.524	0.78	0.98629
	1:1.5	319.4747 ± 3.6	0.09787 ± 0.0035	323.8	313.1031	3.3	0.98439
	1:2	-1.327 ± 0.336	-0.1108 ± 0.00683	93	110.257	18.6	0.95899
	1:2.5	333.045 ± 31.2	0.04659 ± 0.0084	320.7	281.3874	12.3	0.82175
	1:3	252.281 ± 4.86	0.1185 ± 0.00833	275	250.076	9.1	0.93189
Mono-digester Sheep	1:1	293.505 ± 8.39	0.04513 ± 0.002	245.2	245.2421	0.017	0.98892
	1:1.5	122.223 ± 1.7	0.04976 ± 0.0014	105.2	105.5222	0.31	0.99618
	1:2	-0.116 ± 0.049	-0.12328 ± 0.0114	13.03	15.9829	22.6	0.90963
	1:2.5	80.026 ± 0.799	0.07281 ± 0.002	74.5	75.67713	1.58	0.99456
	1:3	45.7378 ± 0.41	0.06215 ± 0.0013	41.65	41.93065	0.67	0.99715
Mono-digester Cow	1:1	268.67 ± 21.02	0.02114 ± 0.0023	154.7	153.353	0.87	0.9889
	1:1.5	321.89 ± 9.743	0.04949 ± 0.003	279	277.4375	0.56	0.98114
	1:2	70245.3 ±2E8	6.01387E-6 ±0.02	20.55	16.89582	17.8	0.74148
	1:2.5	50.37309 ± 0.9	0.07818 ± 0.0037	48	48.16466	0.34	0.98102
	1:3	29.828 ± 0.654	0.0667 ± 0.00355	27.4	27.75916	1.31	0.98141
Co-digester	1:1	-325.17 ± 57.6	-0.0267 ± 0.00316	576	620.932	7.8	0.97912
	1:2	-62.592 ± 6.62	-0.03074 ± 0.0021	141	151.4708	7.4	0.99107
	1:3	0.99107	-0.0336 ± 0.00198	113	121.6772	7.7	0.99171

demonstrated superior predictive stability. The result in the present study was in agreement with previous researches in which that the experimental results fitted well with modified Gompertz modeled values (Ware and Power, 2017). Ratios 1:1, 1:2, and 1:3 produced ultimate yields of 576 L/kg VS, 141 L/kg VS, and 113 L/kg VS respectively, with R_{max} values between 3.9–20.7 L/kg VS.day. Deviations between experimental and predicted yields at day 40 were minimal (Δ = 0.85–2%), and adjusted R^2 exceeded 0.997 in all cases. These results confirm that co-digestion enhances model reliability, reduces error metrics (RMSE < 2 L/kg VS for ratios 1:2 and 1:3), and stabilizes microbial performance. The First-order model produced mixed outcomes. For chicken manure, ratios 1:1 and 1:1.5 yielded realistic B_{max} values (173–319 L/kg VS) with low deviations sheep manure, where ratios 1:1 and 1:1.5 produced accurate fits (Δ < 0.5%), but ratio 1:2 yielded negative B_{max} and Δ = 22.6%. Cow manure showed comparable trends, with realistic fits at ratios 1:1 and 1:1.5 (Δ < 1%), but unrealistic outputs at ratio 1:2 (B_{max} = 70,245 L/kg VS). Additionally, for co-digestion, the First-order model was less reliable. Ratios 1:1 and 1:2 produced negative B_{max} values despite high R^2, while ratio 1:3 yielded a plausible B_{max} = 0.99 L /kg VS but with Δ = 7.7%. These inconsistencies confirm that the First-order model struggles to capture the complexity of multi-substrate interactions, particularly under co-digestion conditions (Leite et al., 2022).

Table 3. Analysis of fit criteria for kinetic models for anaerobic mono and co- digestion

Modified Gompertz model						
Substrate	Statistical indicator	Ratio 1:1	Ratio 1:1.5	Ratio 1:2	Ratio 1:2.5	Ratio 1:3
Mono- digester (Chicken)	RSS	1599.20	8497.44	246.42	13047.41	14849.98
	RMSE	6.32	14.58	2.48	18.05	19.3
	AIC	153.6	220	84.8	236.2	254.8
	BIC	158.7	225.4	90.9	241.3	259.9
Mono- digester (Sheep)	RSS	3161.28	272.85	4.80	116.17	39.43
	RMSE	8.9	2.61	0.35	1.7	0.99
	AIC	163.6	87.2	92.3	48.8	5.4
	BIC	168.7	18.2	23.3	53.5	10.6
Mono- digester (Cow)	RSS	1646.15	5800.49	22.36	156.98	33.48
	RMSE	6.42	12.04	0.75	1.98	0.91
	AIC	154.4	205.2	-17.2	60.4	-1.2
	BIC	159.5	210.3	-12.1	65.5	3.9
Co-digester	RSS	3713.28674	-	147.03	-	148.84
	RMSE	9.64	-	1.92	-	1.93
	AIC	187.2	-	58.0	-	58.4
	BIC	192.3	-	63.1	-	63.5
First order kinetic model						
Substrate	Statistical indicator	Ratio 1:1	Ratio 1:1.5	Ratio 1:2	Ratio 1:2.5	Ratio 1:3
Mono- digester (Chicken)	RSS	1107.12	3738.17	1588.23	33749.90	10101.80
	RMSE	5.26	9.66	6.31	29	15.9
	AIC	137.8	185.2	151.2	273.2	225.2
	BIC	141.2	188.6	154.6	276.6	228.6
Mono- digester (Sheep)	RSS	2196.66	124.38	75.80	88.65	14.56
	RMSE	7.41	1.76	1.38	1.49	0.6
	AIC	164.4	49.2	29.2	35.6	-36.4
	BIC	167.8	52.6	32.6	39	-33
Mono- digester (Cow)	RSS	898.72	4031.53	623.64	133.92	46.25
	RMSE	4.74	10.05	3.95	1.83	1.08
	AIC	128.4	188.4	113.6	52.4	10
	BIC	131.8	191.8	117	55.8	13.4
Co-digester	RSS	29855.54	-	729.41	-	431.49
	RMSE	27.3	-	4.27	-	3.28
	AIC	270.8	-	120.4	-	99.2
	BIC	274.2	-	123.8	-	102.6

In addition, statistical indicators (RSS, RMSE, AIC, and BIC) as shown in (Table 3). For mono-digestion, Gompertz fits consistently produced lower RSS and RMSE values compared to First-order fits. For chicken manure, Gompertz produced RSS values as low as 246 at ratio 1:2 compared to inflated RSS values exceeding 33,000 under the First-order model at ratio 1:2.5, highlighting the latter's tendency toward over-parameterization. RMSE values under Gompertz remained within acceptable ranges (2.48–19.3), while first-order fits often yielded much higher errors (up to 29), particularly under diluted conditions. AIC and BIC scores further confirmed Gompertz's robustness, with values consistently lower and in some cases negative (e.g., cow manure ratios 1:2 and 1:3), indicating exceptionally strong fits due to minimal residual variance. In contrast, first-order models produced inflated AIC/BIC scores (>270) under co-digestion, reflecting poor suitability for complex multi-substrate systems. Sheep manure results reinforced this pattern: Gompertz yielded very low RMSE values (0.35–2.61) and parsimonious AIC/BIC scores, while first-order produced negative or unrealistic Bmax values despite superficially acceptable error metrics. Co-digestion systems provided the clearest evidence of Gompertz superiority, with RSS values below 150, RMSE < 2, and AIC/BIC scores under 60, compared to First-order fits that generated RSS values above 29,000 and inflated error statistics.

For example, chicken manure at ratio 1:1 yielded RMSE = 6.32 under Gompertz versus 5.26 under First-order, but with more realistic B_{max}. Sheep manure at ratio 1:3 yielded RMSE = 0.99 under Gompertz and 0.60 under First-order, yet Gompertz provided physically meaningful outputs. Also, Co-digestion fits under Gompertz consistently produced RMSE < 2 L/ kg VS and AIC/BIC values below 60, confirming strong predictive accuracy also, which supports a conclusion that they are close predictors for model parameters (Das et al., 2025). In contrast, first-order fits for co-digestion yielded inflated RSS values (up to 29,855) and high AIC/BIC scores (>270), reflecting poor model suitability. This aligns with literature reports that co-digestion enhances microbial diversity, balances nutrient ratios, and improves CH_4 yield consistency (Ibro et al., 2022).

The reliability of the Modified Gompertz model observed in this study has been consistently demonstrated in recent investigations on biogas production from animal manures and related substrates. In our work, Gompertz provided realistic yields, short lag phases, and high adjusted R^2 values, particularly under co-digestion conditions. Similar outcomes were reported by Lower et al. (2025), where thermophilic anaerobic digestion of duckweed varieties produced biochemical methane potentials ranging from 205 to 262 ml CH_4/ g VS. Their analysis showed that Gompertz fitted the data with exceptional accuracy, and the absence of a lag phase was attributed to the use of acclimated inoculum. Comparable findings were presented by Almomani and Bhosale (2020), who investigated the co-digestion of agricultural solid wastes with cow dung. A cumulative CH_4 yield of 297.99 L/kg VS was achieved at the optimum 60:40 ratio, and Gompertz was identified as the most reliable model, with R^2> 0.979 and residual deviations below 5%. These results reinforce the conclusion that Gompertz captures microbial adaptation and maximum production rates more effectively than simpler kinetic models. Further evidence was provided by Tian et al. (2020), who modeled biogas production kinetics under heavy-metal stress. Gompertz was shown to outperform Logistic functions, with correlation coefficients exceeding 0.99 and smaller discrepancies between experimental and predicted yields. This supports the robustness of Gompertz even under inhibitory conditions. In contrast, the limitations of the First-order model observed in our study have also been documented elsewhere. Nweke and Nwabanne (2020) reported a First-order constant of 0.095 day^{-1} for plantain peel digestion, with COD removal efficiency of 72.5%. While the model was suitable for simple mono-digestion systems, second-order models failed, highlighting the limitations of simplistic kinetics. Similarly, Lower et al. (2025) reported First-order constants of 0.205–0.285 day^{-1} for duckweed digestion, but emphasized that Gompertz provided superior fits, particularly in capturing lag phases. The CH_4 potentials of 155–323 mL CH_4/gVS were reported by Kafle and Chen (2016), with the First-order model fitting best and deviations below 3%. In our analysis, the Modified Gompertz model produced more accurate predictions, particularly under co-digestion where adjusted R^2 values exceeded 0.997 and deviations were less than 2%. These results indicate that Gompertz modeling is the most reliable framework for complex multi-substrate systems, with co-digestion further enhancing stability and CH_4 yield. The statistical robustness of Gompertz modeling has been further confirmed in recent reviews. Almomani and Bhosale (2020) reported residual deviations (SDR ≤ 5.21) and high predictive accuracy, while Galal et al. (2025) concluded that Gompertz remains the benchmark model due to its ability to minimize residual variance and avoid overfitting. The comparative analysis demonstrated that the Modified Gompertz model was more reliable than the First-order model for predicting biogas kinetics across both mono- and co-digestion systems. Realistic yields, low error metrics, and high adjusted values were consistently produced by Gompertz, particularly under co-digestion conditions where deviations remained below 2%. In contrast, the First-order model, although simpler, failed to capture the complexity of substrate interactions and frequently generated unrealistic or negative yield values.

The superior performance of co-digestion under Gompertz modeling was highlighted by the synergistic stabilization effect of mixing substrates. Variability was reduced, prediction errors were minimized, and statistical robustness was improved. This observation aligns with literature reports indicating that co-digestion enhances microbial diversity, balances nutrient ratios, and improves methane yield consistency. The anomalies observed in mono-digestion, such as extremely high or negative values, further emphasize the need for advanced models when dealing with single substrates prone to variability. From a methodological perspective, the inclusion of multiple statistical indicators (RSS, RMSE, AIC, BIC) strengthened model selection. Gompertz consistently produced lower error metrics and more parsimonious fits. Negative AIC/BIC values observed in some Gompertz fits (e.g., cow manure ratios 1:2 and 1:3) reflected exceptionally strong fits due to very low residual variance. In contrast, First-order fits for co-digestion yielded inflated RSS values (up to 29,855) and high AIC/BIC scores (>270), reflecting poor suitability.

Overall, the findings demonstrate that the Modified Gompertz model is the most suitable framework for biogas research, especially in evaluating co-digestion systems. This study provides a solid foundation for recommending Gompertz modeling in future optimization and scaling efforts to enhance anaerobic digestion processes. Notably, it offers a systematic comparison of chicken, sheep, and cow manures under both mono- and co-digestion conditions, showcasing significant synergistic stabilization among various livestock feedstocks.

Optimization of biogas yield

The regression analysis of the predicted and experimental biogas yields shown in (Figure 7), a nearly perfect linear relationship, with a slope of 0.9958 and an intercept of -1.47. This means that the kinetic model is very similar to what happened in the experiments, with very little bias across the different conditions that were tested.

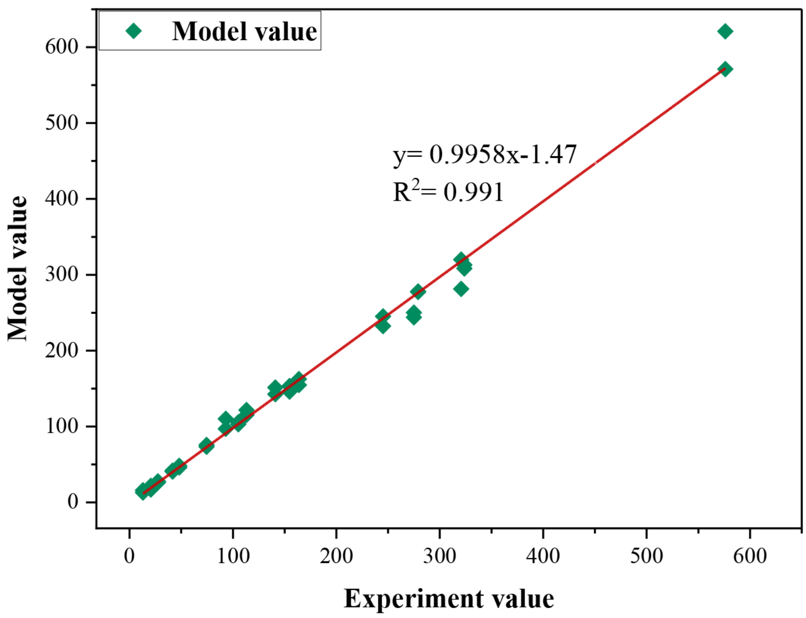

Figure 7. The relationship between the predicted and measured biogas yield

The coefficient of determination ($R^2 = 0.991$) and Pearson's correlation coefficient ($r = 0.996$) show that the model can explain more than 99% of the changes in experimental methane yield. The adjusted R^2 (0.991) further confirms that the regression fit is strong, making sure that the model is still accurate even when the number of predictors is taken into account.

This finding is confirmed by statistical results. The ANOVA findings revealed a highly significant regression ($F = 3924$, $p < 0.001$), indicating that the slope is statistically distinct from zero. The residual sum of squares (RSS ≈ 6219) was low in relation to the overall variance. This indicates that anticipated yields are highly dependent on experimental values rather than being arbitrary approximations. The accuracy of the regression parameters is demonstrated by the small confidence intervals for the intercept (±3.49) and slope (±0.016). These results offer precise criteria for making decisions from an optimization standpoint. Regression-validated kinetic parameters can be used to directly identify conditions that optimize the daily R_{max} while minimizing λ. It is statistically possible to defend as optimal substrate ratios or operational settings that produce high B_{max} with low error metrics (RSS, RMSE, AIC, BIC). Thus, the regression framework makes it possible to forecast performance under untested circumstances, directing the choice of feedstock combinations and retention periods that strike a balance between stability and yield.

The regression-based optimization in this study yielded strong predictive accuracy, with a slope near unity (0.996) and a high coefficient of determination ($R^2 = 0.991$). These results align with earlier work by Li et al. (2018), who showed that exponential and Gompertz models reliably describe organic degradation and methane yield in food waste digestion, and emphasized the role of kinetic modeling in identifying optimal substrate compositions. Similarly, Kafle et al. (2014) demonstrated the superiority of the Modified Gompertz model over first-order kinetics for livestock manure, particularly in capturing lag phases and microbial adaptation. El-Mashad (2013) further highlighted the Cone model as a robust descriptor of cumulative biogas production, underscoring the importance of model choice. Building on these insights, the present study confirms that regression-validated Gompertz modeling provides excellent fit under co-digestion conditions, with low error metrics (RSS ≈ 6219, favorable RMSE, AIC/BIC). Collectively, these findings reinforce regression-driven kinetic modeling as a defensible framework for optimization, bridging experimental data with predictive tools and extending methodological continuity to mesophilic co-digestion systems.

In practical terms, the strong correlation between predicted and experimental data confirms that regression-based kinetic modeling can be used as a reliable optimization tool. By applying these parameters, operators can design digestion systems that achieve rapid microbial adaptation, maximize CH_4 recovery, and minimize variability. This approach provides a scalable pathway for improving anaerobic digestion efficiency, particularly in mesophilic climates such as Erbil City, where natural conditions favor stable microbial activity without additional energy input.

CONCLUSIONS

Pilot-scale anaerobic digestion of livestock manure under ambient conditions yielded distinct performance outcomes across substrates. Chicken manure exhibited the fastest onset of gas generation and the highest cumulative yields under both mesophilic and thermophilic regimes. Cow manure, while less productive overall, showed improved adaptability under mesophilic conditions when moderately diluted. Sheep manure produced intermediate yields, with results strongly influenced by dilution ratio and operating temperature. Co-digestion at a 1:1 ratio produced a pronounced synergistic effect, with cumulative biogas and CH_4 yield reaching 576 L/kg VS and 374 L/kg VS respectively, nearly double that of mono-digestion. Production curves were smoother and process stability was enhanced, confirming the advantage of substrate blending. Kinetic modeling provided further insights into process dynamics. The Modified Gompertz model consistently outperformed the First-order model, delivering robust fits with high adjusted R^2 values (>0.95) and low error indices. Regression analysis confirmed excellent agreement between predicted and experimental yields ($R^2 = 0.991$), establishing the Gompertz model as a reliable predictive tool for biogas optimization. The integration of experimental trials with regression-based modeling established a reproducible framework for optimizing biogas yield from livestock waste.

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